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INVESTIGATION OF TRANSIENTS IN AN
ANALOGUE CIRCUIT FOR AN IGNITRON
MOTOR CONTROL SYSTEM

CARLETON FANTON BRYANT, JR.
CLAYTON RAND ADAMS

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INVESTIGATION OF TRANSIENTS IN AN ANALOGUE CIRCUIT
FOR AN IGNITRON MOTOR CONTROL SYSTEM

by

Carleton Fanton Bryant, Jr., Lieutenant Commander, U. S. Navy

B.S., Massachusetts Institute of Technology, 1943.

Clayton Rand Adams, Lieutenant (j.g.), U. S. Navy

B.S., U. S. Naval Academy, 1947.

Submitted in Partial Fulfillment
of the Requirements for the
Degree of Naval Engineer

From the
Massachusetts Institute of Technology
1952

ABSTRACT

INVESTIGATION OF TRANSIENTS IN AN ANALOGUE CIRCUIT FOR AN IGNITRON MOTOR CONTROL SYSTEM

by

Carleton F. Bryant, Jr.
Clayton R. Adams

Submitted to the Department of Naval Architecture and Marine Engineering on 15 May 1952 in partial fulfillment of the requirements for the degree of Naval Engineer.

This thesis is a continuation of an investigation of transient speed characteristics of an ignitron-fed d-c motor undertaken by P. N. Heller, which consisted of actual tests on a 15 horsepower motor with a 3-phase ignitron rectifier and the development of two analytical methods of predicting the transient behavior. Because of the involved calculations required for the analytical methods and the difficulties of conducting full scale tests, it was proposed that an analogue circuit be used for future tests of this type.

The primary purpose of this investigation was the construction, testing, and evaluation of this analogue circuit. It was found that the usefulness of the analogue was limited by a transient current occurring during discontinuous conduction at the termination of each current pulse causing the analogue to misrepresent actual ignitron and motor performance in the boundary region between continuous and discontinuous conduction.

It was also found during the course of the investigation, however, that the transient response of the system could be predicted with reasonable accuracy using an exponential time constant approach which involves considerably less computational work than the previous methods.

Thesis Supervisor:	Alexander Kusko
Title:	Assistant Professor of Electrical Engineering

ABSTRACT

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FOR AN IGNITION MOTOR CONTROL SYSTEM

by

Carlton F. Bryant, Jr.
Clayton R. Adams

Submitted to the Department of Naval Architecture and
Marine Engineering on 15 May 1952 in partial fulfillment
of the requirements for the degree of Naval Engineer.

This thesis is a continuation of an investigation of
transient speed characteristics of an ignition-test 4-c
motor undertaken by I. W. Heller, which consisted of actual
tests on a 15 horsepower motor with a 3-phase ignition recti-
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calculations required for the analytical methods and the
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gation, however, that the transient response of the system
could be predicted with reasonable accuracy using an
exponential time constant approach which involves consider-
ably less computational work than the previous methods.

Thesis Supervisor: Alexander Kusko
Assistant Professor of
Electrical Engineering

Cambridge, Massachusetts

May 15, 1952

Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer, we submit herewith a thesis entitled "Investigation of Transients In An Analogue Circuit For An Ignitron Motor Control System".

Respectfully,

Carleton F. Bryant, Jr.
Lieutenant Commander
U. S. Navy

Clayton R. Adams
Lieutenant, (j.g.)
U. S. Navy

Cambridge, Massachusetts

May 12, 1932

University of the Pacific
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the
Degree of Naval Engineer, we submit herewith a thesis
entitled "Investigation of Transients in an Analogous
Circuit for an Ignition Motor Control System".

Respectfully,

Clayton W. Adams
Lieutenant Commander
U. S. Navy

Clayton W. Adams
Lieutenant (j.g.)
U. S. Navy

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The authors wish to express their appreciation to Professor Alexander Kusko for his advice and encouragement; and to Mr. P. N. Heller for his helpful suggestions.

CONFIDENTIAL
JAN 19 1954

APPENDIX

The authors wish to express their appreciation to
Professor Alexander Lurko for his advice and encouragement;
and to Mr. V. V. Belter for his helpful suggestions.

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INTRODUCTION

Direct-current motors connected to a-c power supplies through grid-controlled rectifiers are being used at the present time for power applications requiring a wide range of speed control. Single-phase full-wave thyatron rectifiers are generally used for fractional horsepower installations, while three-phase ignitron rectifiers are used for the higher ratings.

Speed control of the motor is obtained by changing the firing angle of the rectifier. The system is a substitute for a Ward-Leonard type of control with the rectifier replacing the d-c generator, although the governing action of the ignitrons or thyratrons is probably more analogous to throttling in a mechanical power device.

While the steady state operation of this type of electronic drive has been rather fully investigated,^{1) 2) 3)} relatively little has been published dealing with the transient behavior of the system for major changes of speed or load.

-
- 1) Vedder, E. H. and Puchlowski, K. P., "Theory of Rectifier D-C Motor Drive", AIEE Trans., 62, 1943, pages 863-870.
 - 2) Schmidt, A. and Smith, W. P., "Operation of Large D-C Motors from Controlled Rectifiers", AIEE Trans., 67, 1948, pages 679-683.
 - 3) Chute, G. M., "Electronic Motor and Welder Controls", McGraw-Hill Book Co., 1951, pages 191-202, 226-277.

INTRODUCTION

Direct-current motors connected to a-c power supplies through this-controlled rectifiers are being used in the greatest time for power applications requiring a wide range of speed control. This class of full-wave thyristor rectifiers are generally used for fractional horsepower installations, while three-phase thyristor rectifiers are used for the higher ratings.

Speed control of the motor is obtained by changing the firing angle of the rectifier. The speed is a substantial function of the firing angle of the thyristor. The thyristor type of control with the thyristor rectifier in the d-c converter, affords the governing action of the inverter of thyristor is probably more analogous to operating in a conventional power device.

While the steady state operation of this type of electronic drive has been rather fully investigated, relatively little has been published dealing with the transient behavior of the system for major changes of speed or load.

1) Vukobratovic, D. and Vukobratovic, D. L., "Theory of Thyristor d-c Motor Drive", IEEE Trans., 52, 1967, pages 667-670.

2) Vukobratovic, D. and Vukobratovic, D. L., "Operation of Large d-c Motors from Thyristor Rectifiers", IEEE Trans., 52, 1967, pages 672-681.

3) Cramer, D. L., "Electronic Motor and Field Controls", McGraw-Hill Book Co., 1961, pages 191-202, 258-277.

This thesis is a continuation of an investigation of transient speed characteristics of an ignitron-fed d-c motor undertaken by P. N. Heller,⁴⁾ which consisted of actual tests on a 15 horsepower motor with a 3-phase ignitron rectifier and the development of two analytical methods of predicting the transient behavior. Because of the involved calculations required for the analytical methods and the difficulties of conducting full scale tests, it was proposed that an analogue circuit be used for future tests of this type.

The primary purpose of this investigation was the construction, testing, and evaluation of this analogue circuit. It was also found during the course of the investigation, however, that the transient response of the system could be predicted with reasonable accuracy using an exponential time constant approach which involves considerably less computational work than the previous methods. In the presentation which follows, these two aspects will be considered separately.

4) Heller, P. N., "Transient Speed and Armature Current Characteristics of an Ignitron-Fed D-C Motor". M.I.T. E.E. Dept. Thesis 1951.

This thesis is a continuation of an investigation of
 transient speed characteristics of an induction motor
 motor resistance of 1.5 ohms, which consisted of several
 tests on a 15 horsepower motor with a 2-phase induction motor.
 Then was the development of an analytical method of pre-
 dicting the transient behavior. Because of the involved
 calculations required for the analytical method and the
 difficulty of determining the exact value of the
 time constant it was decided to use the Laplace method of this
 type.

The primary purpose of this investigation was the com-
 parative, testing, and evaluation of this analytical circuit.
 It was also found during the course of the investigation,
 however, that the transient response of the system could be
 predicted with reasonable accuracy using an experimental
 time constant approach which involves considerably less
 computational work than the previous method. In the pre-
 sentation which follows, these two methods will be compared
 separately.

A. J. Heller, E. E., "Transient Speed and Torque
 Characteristics of an Induction
 Motor," M.S. Thesis, M.I.T., Boston, 1951.

DERIVATION OF THE ANALOGUE CIRCUIT

The analogue circuit used for this investigation was intended to represent the 15 horsepower separately excited d-c shunt motor and the 3-phase ignitron rectifier used by Heller in obtaining experimental speed response curves for step changes in firing angle.⁴⁾ The name-plate data of these units together with the generator used for loading the system is given in Appendix A.

Mechanical-Electrical Equivalents.

The actual circuit studied is shown in Figure 1. The justification for the electrical representation of the mechanical motor characteristics may be shown by the following analysis in which the air gap flux is considered constant (constant shunt field current with armature reaction neglected).

The electrical relationship is

$$v_t = L_a \frac{di_a}{dt} + i_a R_a + V_b + e_b \quad (1)$$

where v_t = voltage applied to armature circuit.

i_a = armature current

e_b = counter-emf (voltage between points d and e of Figure 1.)

V_b = brush drop (absorbed with tube drop in circuit)

R_a = armature resistance

L_a = armature inductance

t = time

The instantaneous air-gap torque of the motor rigidly coupled to a load with linear characteristics is given by

MECHANICAL-ELECTRICAL ANALOGY

The analogous circuit used in this investigation was intended to represent the 12 components separately existing in a motor and the 3-phase lighting system used by it. It is obtained experimentally upon a basis of step changes in time units. The same-phase base of time units together with the parameter used for isolating the system is given in Appendix A.

Mechanical-Electrical Analogy

The general circuit studied is shown in Figure 1. The induction for the electrical representation of the mechanical motor characteristics may be shown by the following analysis in which the air gap flux is considered constant (constant means this current with structure remains constant).

The electrical relationship is

$$V_t = I_a \frac{d\phi}{dt} + I_a R_a + V_b + V_c \quad (1)$$

where V_t = voltage applied to structure circuit.

I_a = armature current

V_b = counter-EMF (voltage between poles b and c of Figure 1.)

V_c = brush drop (assumed with this drop in circuit)

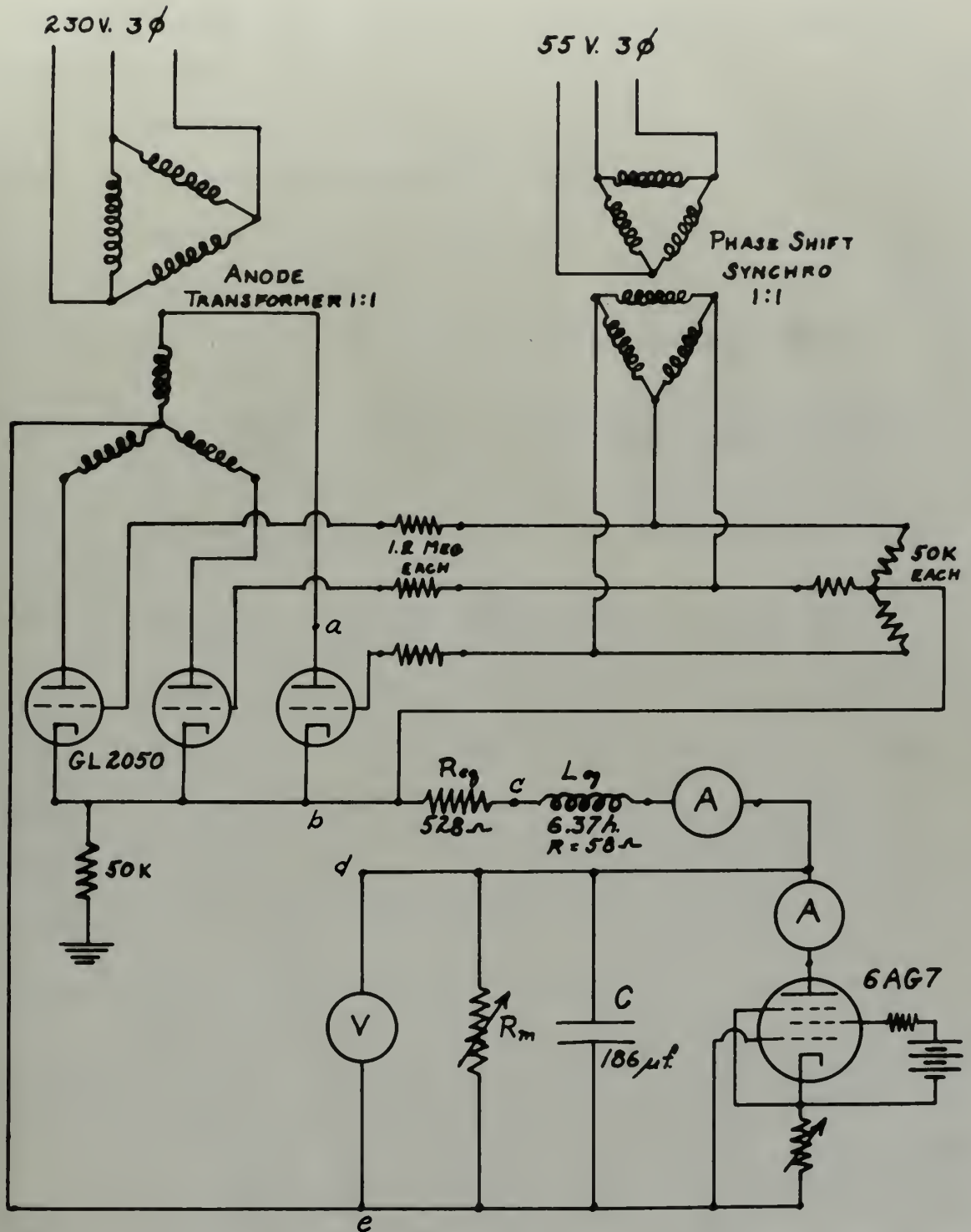
R_a = armature resistance

I_a = armature current

t = time

The instantaneous air-gap torque of the motor is

coupled to a load with linear characteristics is given by



ANALOGUE CIRCUIT DIAGRAM
FIG 1

$$(J_L + J_a) \frac{dw}{dt} + (B_L + B_a) w + T_L + T_a = T_{ag} \quad (2)$$

Here

w = Motor speed

T_{ag} = instantaneous air-gap torque

J_L = load inertia

J_a = armature inertia

B_L = component of load torque which is proportional to speed.

B_a = coefficient of those components of motor friction, windage, and core-loss torque which are proportional to speed.

T_L = component of load torque which is constant at all speeds.

T_a = hysteresis and other rotational loss torques of the motor which are constant at all speeds.

The counter-emf of the motor is directly proportional to the product of motor speed and air-gap flux; the air-gap torque is directly proportional to the product of armature current and air-gap flux. It is assumed that the air-gap flux of the shunt motor is constant and that hence

$$e_b = k_1 w \quad (3)$$

$$T_{ag} = k_2 i_a \quad (4)$$

The electro-mechanical coupling constants k_1 and k_2 can be determined from test or design data of the motor.

Equation (2) can be translated into electrical terms by substituting $\frac{e_b}{k_1}$ for w and $k_2 i_a$ for T_{ag} , giving,

$$\frac{J_L + J_a}{k_1 k_2} \frac{de_b}{dt} + \frac{B_L + B_a}{k_1 k_2} e_b + \frac{T_L + T_a}{k_2} = i_a \quad (5)$$

This last equation describes the behavior of the parallel circuit between points d and e of Figure 1 if

$$C = \frac{J_L + J_a}{k_1 k_2} \quad (6)$$

$$R_m = \frac{k_1 k_2}{B_L + B_a} \quad (7)$$

$$i_{L1} = \frac{T_L + T_a}{k_2} \quad (8)$$

The 6AG7 pentode with cathode bias represents the current source i_{L1} while the GL 2050 thyratrons simulate the ignitron rectifier.

Scale Ratios.

The scale ratios between motor and analogue are most easily obtained by expressing voltages, currents and impedances as per unit values. Base values taken were the anode transformer secondary rms voltage and the armature current. No attempt was made to change the time scale since the only three phase power available was 60 cycle.

Choice of the Analogue Circuit Elements.

In order to keep the effects of tube voltage drop of the same order of magnitude in both the analogue circuit and the actual rectifier, 220 volts was used for the anode transformer secondary voltage. Type GL 2050 thyratrons were chosen for the rectifier since they have the necessary inverse voltage and average and peak current ratings for the circuit and were readily available. The current level of the circuit was

This last equation determines the behavior of the potential
at points between points A and B of Figure 1 if

$$(6) \quad \frac{d^2 V}{dx^2} + \frac{V}{L^2} = 0$$

$$(7) \quad \frac{dV}{dx} = 0 \quad \text{at } x = 0$$

$$(8) \quad V = 0 \quad \text{at } x = L$$

The only potential with nodes also represents the
correct source V_1 while the 0.1500 represents approximate
the function coefficient.
Final Notes.

The scale factor between motor and analysis are most
easily obtained by representing voltages, currents and impedances
as per unit values. These values taken were the same from
former secondary has voltage and the primary circuit. No
effort was made to change the line scale since the only three
phase power available was 60 cycle.
Choice of the Analysis Circuit Diagram.

In order to keep the effects of node voltage drop at
the same order of magnitude in both the analysis circuit and
the actual rectifier, 250 volts was used for the node trans-
former secondary voltage. The 0.1500 tap ratio was chosen
for the rectifier since they save the necessary inverse voltage
and average and peak current ratings for the circuit and were
readily available. The current level of the circuit was

determined by the capacity of available vacuum tubes used for the current source. By using two 6AG7 pentodes in parallel it was possible to pass about 60 milliamperes, giving a ratio of actual motor current to analogue current of about 1000 to 1.

The impedance level of the analogue was actually determined by the iron-core reactor used to represent the armature inductance and the anode transformer leakage inductance. Available units were tested by obtaining magnetization curves with 60 cycle current up to a value of about 70 ma. The unit chosen indicated no saturation up to this point. A laboratory capacitor unit was adjusted and tested in the same manner to give the desired value of capacitance for the circuit. The motor and analogue circuit element values are listed in Table I.

Transformers.

The anode transformer used was a bank of three 5 KVA 110/220 to 110/220 single phase power transformers. Since the leakage inductance and resistance of these units is negligible on the basis of the analogue circuit the equivalent leakage inductance and resistance of the actual rectifier transformer has been lumped with the armature resistance and inductance of the motor. This is not strictly accurate since the transformer equivalents should appear in the input circuit to the thyratrons, but the results are not considered to be affected appreciably by this approximation.

The phase shift circuit was supplied by reduced voltages taken from a bank of transformers similar to the anode trans-

TABLE I

MOTOR AND ANALOGUE CIRCUIT ELEMENT VALUES

Element	Motor	Per Unit	Analogue
Anode Transformer Secondary Voltage	232 v.	1.00	220 v.
Motor-Rated Current	56 a.	1.00	0.0518 a.
Armature Resistance (60 cycle a-c)	0.484 Ω	0.117	495 Ω
Transformer Resistance	0.089 Ω	0.0215	91 Ω
Armature Inductance	5.84×10^{-3} h.	1.41×10^{-3}	5.97 h.
Transformer Leakage Inductance	0.39×10^{-3} h.	0.094×10^{-3}	0.40 h.
Equivalent Motor Inertia	0.190 + 5% f.	0.79	186×10^{-6} f.

Motor data taken from reference 4.

Table 1

Table 1. Results of the analysis of variance

Source	df	SS	MS	F	P
Between groups	3	1.25	0.42	1.25	0.35
Within groups	12	1.25	0.10		
Total	15	2.50			
Between groups	3	1.25	0.42	1.25	0.35
Within groups	12	1.25	0.10		
Total	15	2.50			
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Total	15	2.50			

1. The first two rows of the table show the results of the analysis of variance for the first two rows of the data.

2. The third row of the table shows the results of the analysis of variance for the third row of the data.

3. The fourth row of the table shows the results of the analysis of variance for the fourth row of the data.

4. The fifth row of the table shows the results of the analysis of variance for the fifth row of the data.

5. The sixth row of the table shows the results of the analysis of variance for the sixth row of the data.

6. The seventh row of the table shows the results of the analysis of variance for the seventh row of the data.

7. The eighth row of the table shows the results of the analysis of variance for the eighth row of the data.

8. The ninth row of the table shows the results of the analysis of variance for the ninth row of the data.

9. The tenth row of the table shows the results of the analysis of variance for the tenth row of the data.

10. The eleventh row of the table shows the results of the analysis of variance for the eleventh row of the data.

11. The twelfth row of the table shows the results of the analysis of variance for the twelfth row of the data.

12. The thirteenth row of the table shows the results of the analysis of variance for the thirteenth row of the data.

13. The fourteenth row of the table shows the results of the analysis of variance for the fourteenth row of the data.

14. The fifteenth row of the table shows the results of the analysis of variance for the fifteenth row of the data.

former. The phase shift transformer was a small synchro rated at 96/96 volts. This was provided with a dial and calibrated to read phase shift directly by comparing the output with the thyatron supply voltage on an oscilloscope. Observation indicated that the phase shift as read from the synchro dial was accurate within about two degrees.

...the power of the transmitter was a small ...
...at 100/50 volts. This was provided with a dial and
...calibrated to read power with difficulty by comparing the
output with the standard supply voltage on an oscillo-
scope. Observation indicated that the phase shift was small
from the synchro dial was accurate within about two degrees.

OPERATION OF THE ANALOGUE CIRCUIT

Current Cutoff Transient.

The principle difficulty experienced with the analogue circuit occurred during discontinuous conduction. When each pulse of current cut off an underdamped current oscillation occurred which had a peak value of up to 10% of the current pulse itself and extended for about one-sixth cycle of the applied voltage. This resulted in unstable operation of the circuit in the region between continuous and discontinuous conduction. No means was found of appreciably reducing the magnitude of the oscillation, but by introducing a 50,000 ohm resistor in the ground circuit, it was damped out much more rapidly. It was not found possible however, to eliminate the resulting instability of the circuit.

Steady-State Characteristics.

Tests were made with the circuit to determine both the steady-state and transient performance. Steady-state curves of counter-emf versus armature current were obtained for various firing angles by reading the direct current in the circuit with a d'Arsonval type milliammeter and the voltage between points d and e on Figure 1 with a d-c voltmeter. The values obtained are shown on Figure 2 reduced to a per unit basis and corrected for variations in supply voltage.

Transient Characteristics.

The transient behavior of the counter-emf (speed) for step changes in firing angle was investigated by obtaining

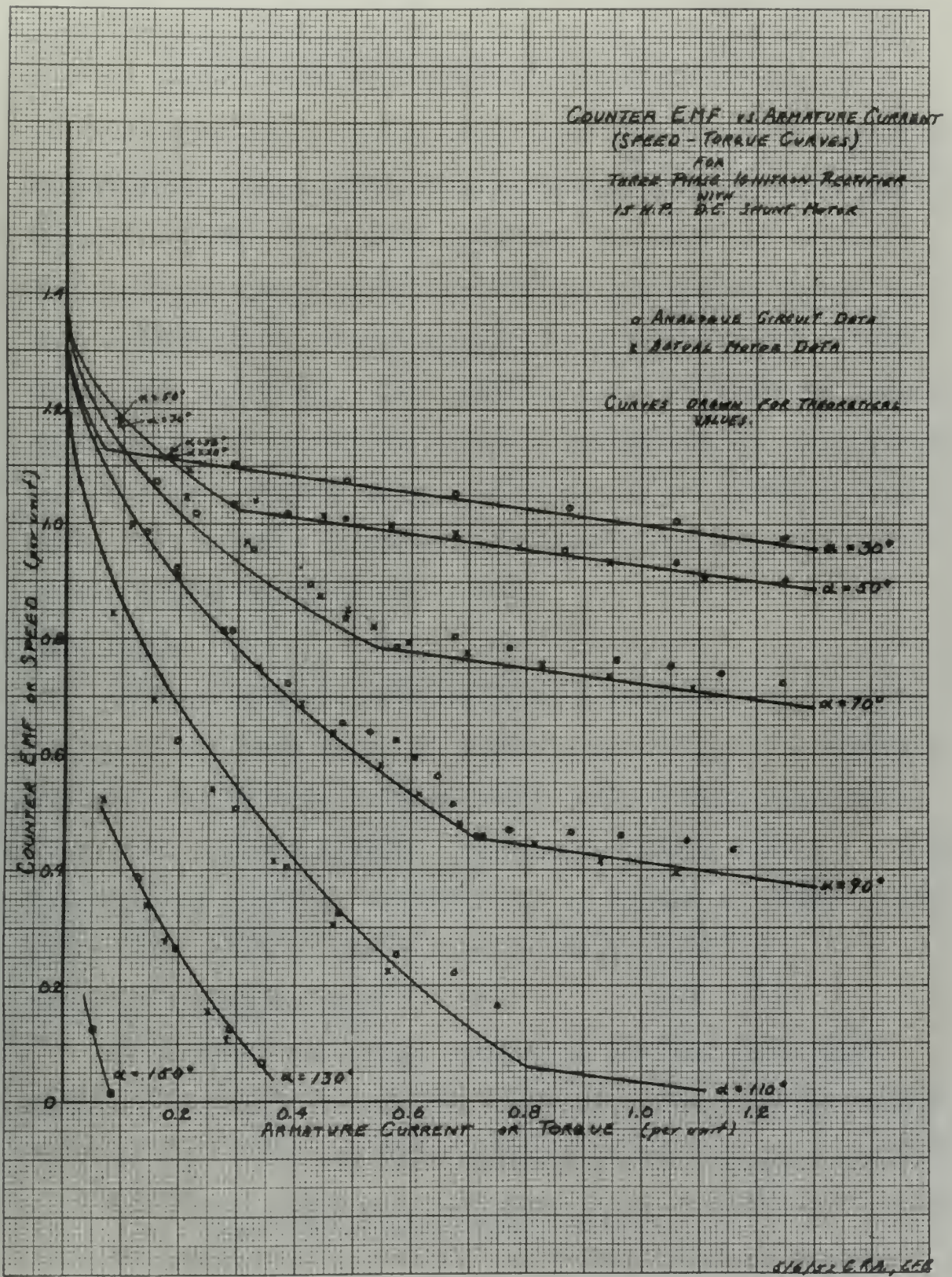


FIG 2

overall response times to reach 95% of the final value for conditions simulating those under which corresponding data for the actual motor was previously obtained by Heller.⁴⁾ The step changes in firing angle were introduced by manually rotating the phase shift synchro between stops to give the desired angles. The speed transient was observed on an oscilloscope connected between points d and e on Figure 1, with a calibrated sweep frequency to give about ten traces during the transient. The response time was obtained merely by counting the number of traces required for the counter-emf to reach the required value and hence may be somewhat inaccurate. The results of these measurements are compared with Heller's data in Table II.

Wave Forms.

Photographs were obtained with a polaroid oscilloscope camera of the wave forms of current, motor terminal voltage, and plate to cathode voltage under various operating conditions with the oscilloscope connected between points b and c, b and e, and a and b, respectively on Figure 1. These photographs are shown on Figures 3, 4, and 5.

Comparative Motor Tests.

In order to determine whether or not the instability found in the operation of the analogue circuit in the region between continuous and discontinuous conduction also occurred with the actual motor and rectifier, tests were made to determine the steady-state counter-emf versus armature current curves for the actual unit. The counter-emf, E_b , was deter-

Overall response time is seen 90% of the time for conditions existing under which conditions are for the same motor and speed as obtained in Figure 1. The way changes in these angles were followed by rotating the phase shift between them to give the desired angles. The speed between was observed on an oscilloscope connected between points a and b in Figure 1, with a calibrated sweep frequency to give some time during the transient. The response time was obtained by counting the number of lines required for the counter to reach the desired value and hence can be converted into rate. The results of these measurements are shown in Table II. Heller's data in Table II.

Wave Form.

Photographs were obtained with a Goldschmidt oscilloscope camera of the wave form of current, motor terminal voltage, and phase to motor voltage under various operating conditions with the oscilloscope connected between points b and c, b and e, and a and e, respectively in Figure 1. These graphs are shown in Figures 3, 4, and 5.

Comprehensive Motor Tests.

In order to determine whether or not the instability found in the operation of the motor circuit in the region between excitation and demagnetization also occurred with the motor motor and generator, tests were made to determine the steady-state unbalanced voltage between the output for the motor and the generator. The counter-act, b, was observed.

TABLE II

TRANSIENT RESPONSE TIME COMPARISON DATA

Run	<u>Speed (rpm)</u>		<u>Angle (degrees)</u>		Motor Response Time (Sec.)	Analogue Response Time (Sec.)
	Initial	Final	Initial	Final		
J-1	902	320	96	135	11.2	11
J-2	460	742	127	109	4.0	4.7
K-1	205	430	137	121	3.2	3.8
K-2	728	465	97	120	4.0	4.2
L-1	490	210	106	132	2.3	3.0
L-2	365	590	118	97	1.4	1.9
L-3	625	810	96	72	1.0	0.2
L-4	842	580	66	102	2.0	1.7
M-1	460	672	105	81	1.1	1.2
M-2	710	501	78	102	1.7	1.7
M-3	460	565	---	97	1.2	1.3
M-4	0	255	180	126	1.7	2.0
N-1	408	825	105	62	0.32	0.32
N-2	830	388	60	109	1.6	1.6
P-1	715	885	70	52	0.34	0.3
P-2	770	410	67	101	1.5	1.2
P-3	880	680	52	74	0.57	0.7
Q-1	535	750	85	65	0.26	0.25
Q-2	875	778	51	63	0.30	0.20

Motor data taken from reference 4.

Table 11

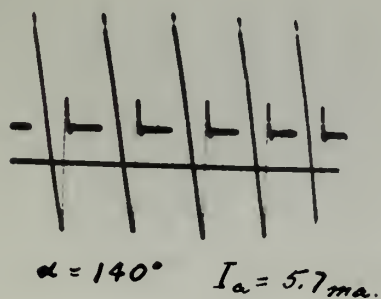
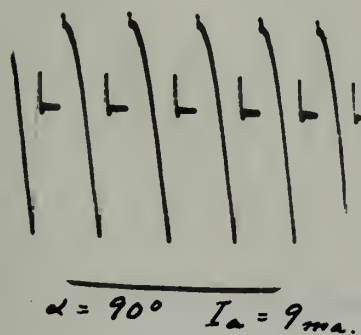
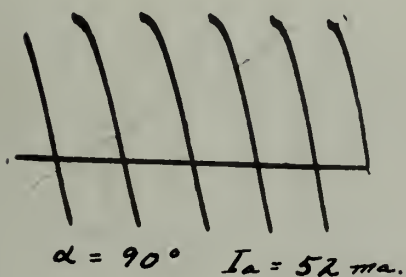
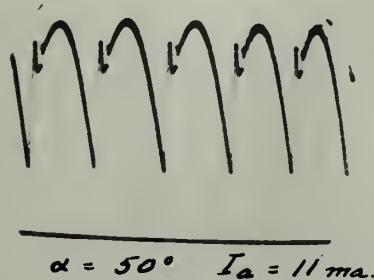
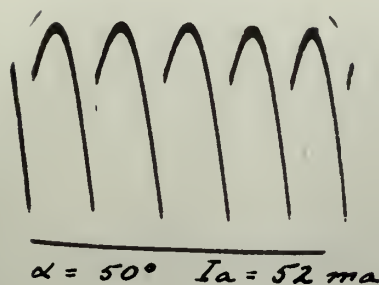
TEST RESULTS FOR THE 1954-55 SEASON

Run	Weight (lb)		Length (inches)		Time (seconds)	
	Initial	Final	Initial	Final	Initial	Final
1-1	305	305	90	335	11.4	11
1-2	400	345	345	100	11.0	4.5
1-3	500	430	415	181	3.5	0.8
1-4	385	405	95	150	4.0	4.5
1-5	400	510	100	135	3.2	3.0
1-6	305	300	119	85	1.4	1.9
1-7	055	010	80	25	1.0	0.5
1-8	045	080	00	105	4.0	1.5
1-9	100	075	105	81	1.1	1.5
1-10	310	301	30	105	1.5	1.8
1-11	100	005	---	05	1.2	1.3
1-12	0	005	100	150	1.5	5.0
1-13	400	005	105	05	0.50	0.35
1-14	070	300	00	100	1.0	1.0
1-15	315	085	50	25	0.30	0.2
1-16	110	470	05	101	1.5	1.5
1-17	080	000	05	35	0.25	0.5
1-18	225	200	05	05	0.50	0.55
1-19	035	330	05	05	0.20	0.50

Notes: 1. All tests were conducted on a standard test track.

FIG 3 — ARMATURE VOLTAGE WAVEFORM

ANALOGUE



MOTOR

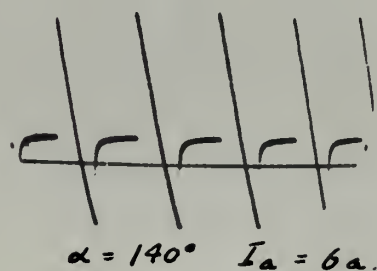
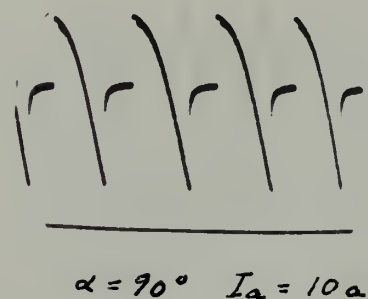
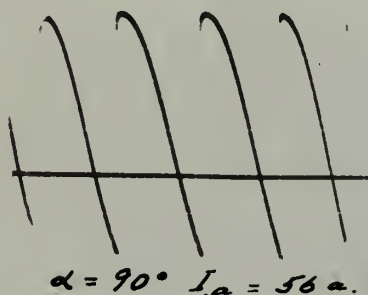
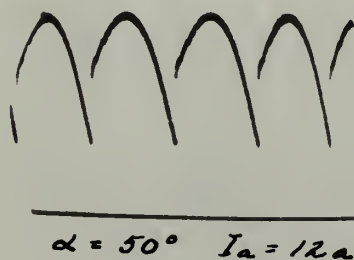
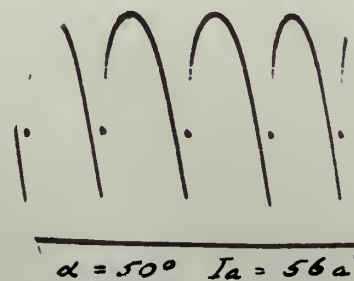
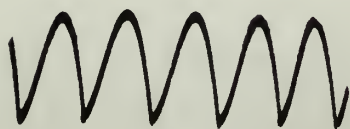
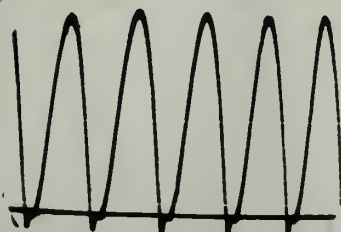


FIG 4 - ARMATURE CURRENT WAVEFORM

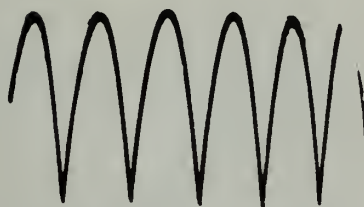
ANALOGUE



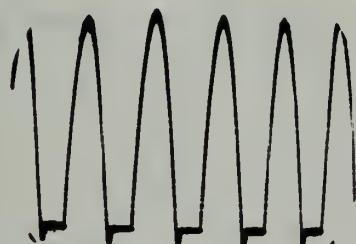
$$\alpha = 50^\circ \quad I_a = 52 \text{ ma.}$$



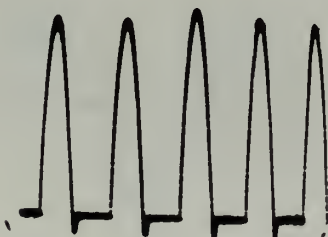
$$\alpha = 50^\circ \quad I_a = 11 \text{ ma.}$$



$$\alpha = 90^\circ \quad I_a = 52 \text{ ma.}$$



$$\alpha = 90^\circ \quad I_a = 9 \text{ ma.}$$

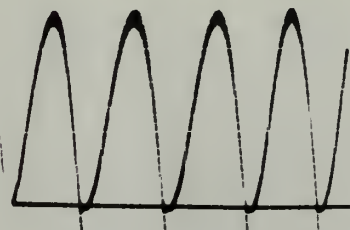


$$\alpha = 140^\circ \quad I_a = 5.7 \text{ ma.}$$

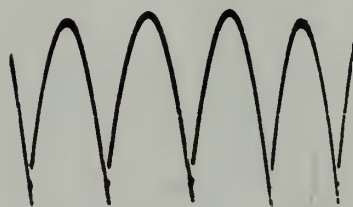
MOTOR



$$\alpha = 50^\circ \quad I_a = 56 \text{ a.}$$



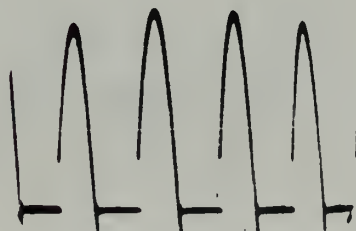
$$\alpha = 50^\circ \quad I_a = 12 \text{ a.}$$



$$\alpha = 90^\circ \quad I_a = 56 \text{ a.}$$



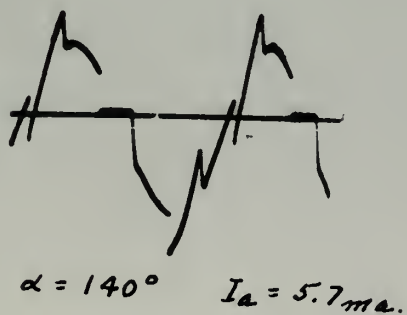
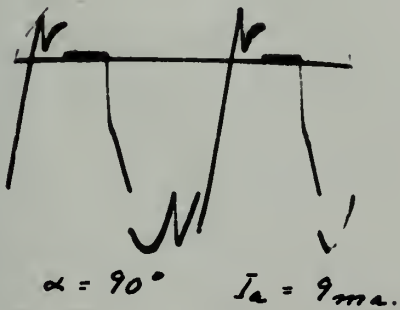
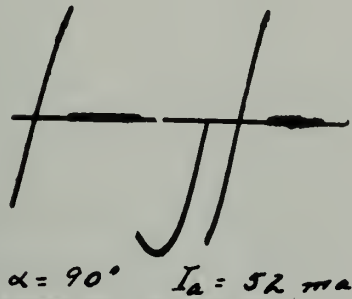
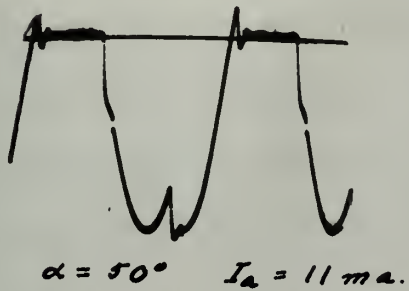
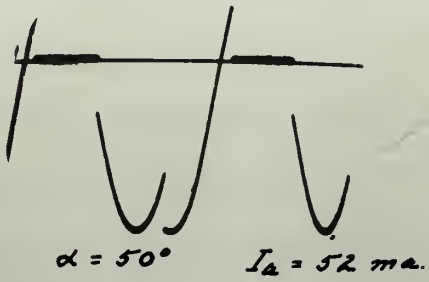
$$\alpha = 90^\circ \quad I_a = 10 \text{ a.}$$



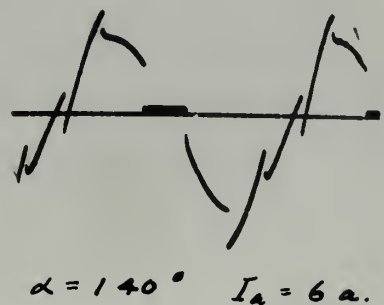
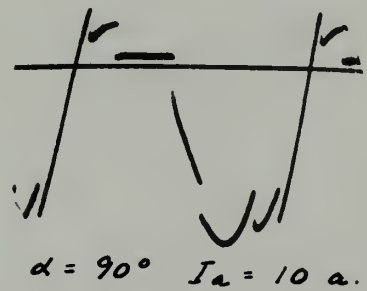
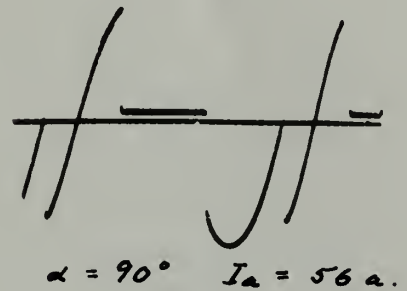
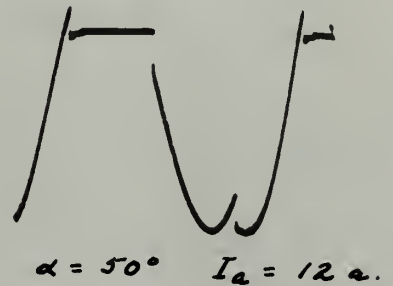
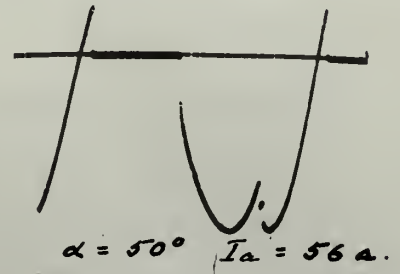
$$\alpha = 140^\circ \quad I_a = 6 \text{ a.}$$

FIG 5 — PLATE TO CATHODE VOLTAGE WAVEFORM

ANALOGUE



MOTOR



mined by measuring the direct motor terminal voltage and subtracting the direct voltage drop due to the measured direct armature current. The points obtained are shown on Figure 2 reduced to a per unit basis and corrected for variations in supply voltage. These tests indicated no instability and no oscillation of the armature current when the pulses cut off during discontinuous conduction. Wave forms were photographed for this unit under conditions corresponding approximately to those used with the analogue circuit and are shown on Figures 3, 4 and 5.

mined by measuring the direct motor terminal voltage and subtracting the direct voltage drop due to the measured direct armature current. The points obtained are shown in Figure 2 reduced to a per unit basis and corrected for variations in supply voltage. These have indicated no instability and no oscillation of the generator output when the pulses are off during synchronous commutation. Wave forms were photographed for data only under conditions corresponding approximately to those used with the analysis circuit and are shown in Figures 3, 4 and 5.

ANALYSIS OF ANALOGUE CIRCUIT PERFORMANCE

Wave Forms.

Comparison of the oscilloscope photographs of the wave forms of armature voltage, armature current and plate to cathode voltage of Figures 3, 4, and 5, indicates that the basic steady-state phenomena of the system are properly represented in the analogue circuit. The armature voltage wave forms for discontinuous conduction show up the major difference encountered, the behavior of the circuit when each current pulse cuts off. The cutoff transient in the current wave forms is not very apparent in these photographs, but analogue armature voltage shows the same underdamped oscillation as it returns to the average value. The similar voltage in the motor does not oscillate and indicates an overdamped behavior. The current overshoot observable on the actual ignitron pulses is probably the result of the time required for deionization. Another point of interest is the abrupt rise at the beginning of the ignitron current pulse under certain conditions which could be caused by a capacitive effect in the leads to the motor or in the housing of the motor itself.

Performance Curves.

The counter-emf versus armature current curves shown on Figure 2 for both the motor and analogue indicate fairly good correlation between the two units. The curves as actually plotted show the theoretical performance and were obtained by

• **0.7107 29417**

Comparison of the oscillations appearing in the wave
forms of the two voltages, measured at the same time in
certain voltage of figures 3, 4, and 5, indicates that the
high steady-state component of the system is properly repre-
sented in the analysis. The transient voltage wave
forms for dissimilar conditions show no major differ-
ence indicated, the behavior of the circuit when such current
takes place. The effect of the transient in the circuit wave
forms is not very apparent in these photographs, but analysis
of the voltage shows the same undamped oscillation as
it refers to the steady value. The similar voltage in the
motor does not oscillate and indicates an overcurrent behavior.
The current overcurrent behavior on the actual circuit pulse
is probably the result of the time required for the
switching point of interest in the supply rise at the beginning
of the initiation current pulse under conditions which
could be caused by a capacitive effect in the leads to the
motor or in the housing of the motor itself.

• REVIEW QUESTIONS

The number-101 series structure current driver shown on Figure 2 for both the color and monochrome indicators fairly good correlation between the two units. The driver is actually plotted with the energized; however, and was obtained by

the methods described on page 49. The experimental points for the motor and analogue show good correlation with each other and with the theoretical curves, considering the fact that the means used for setting the phase shift angles were not accurate within at least 2° .

The non-linear portions of the curves at low armature currents or low voltage are the regions of discontinuous conduction while the straight-line portions to the right are for continuous conduction. The motor performance curves for which counter-emf was obtained by subtracting calculated armature voltage drop from the observed applied direct voltage are fairly flat for high currents; but if speed had been plotted directly, a rising characteristic would have been obtained because of armature reaction.

The analogue curves, on the other hand, break away from both the theoretical and actual motor curves in the boundary region between discontinuous and continuous conduction. This is especially noticeable at firing angles of 90° and 110° , although the condition was found to a certain degree at all angles. The operation of the circuit was actually unstable in these regions, the average current oscillating with a frequency of about 1 cycle per second by as much as 20% and the counter-emf by about 5%.

Observations of the current pulses on an oscilloscope indicated that this was caused by the current cutoff transient mentioned previously. As the load current was increased

The results described in para 16. The experimental points for the motor and machine show good correlation with each other and with the theoretical curve. Considering the fact that the motor used for testing the pump with angles were not accurate within at least 2%.

The non-linear portion of the curve at low currents consists of low voltage and the regions of discontinuous conduction while the straight-line portion at low light are for continuous conduction. The motor performance curves for which number-1 was obtained by substitution of calculated armature voltage from the observed applied direct voltage are fairly flat for high currents; but it bends and soon plotted directly, a rising characteristic which was soon obtained because of pressure reaction.

The engine curves, on the other hand, break away from both the theoretical and actual motor curves in the low-voltage region between discontinuous and continuous conduction. This is especially noticeable at firing angles of 30° and 35°, although the condition was found to be certain degree at all angles. The operation of the circuit was normally unstable in these regions, the average current coefficient with a frequency of about 1 cycle per second up to about 10% and the number-1 by about 2%.

Observations of the current pulses on an oscilloscope indicated that this was caused by the current itself becoming unstable previously. As the load current was increased

gradually during discontinuous conduction, the start of a current pulse would break abruptly between peaks of the cutoff transient from the preceding pulse. For a load at which the pulse first broke away, both conditions were unstable and the pulse initiation would break back and forth between the two peaks as shown in Figure 6 (a) resulting in average current oscillations in the circuit. With further increase in loading, oscillations ceased with the pulse originating from a peak of the transient as shown by Figure 6 (b); but this condition resulted in the offset points on the performance curves. At a slightly higher load, the break occurred as indicated in Figure 6 (c), again causing oscillations in the circuit until the load was sufficient to give full continuous conduction.

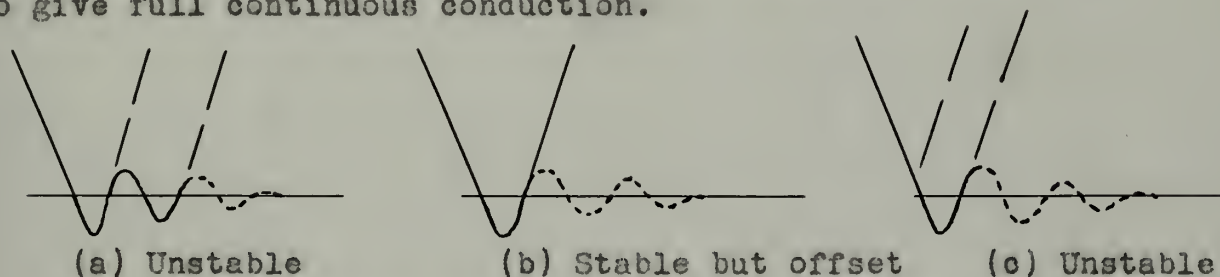


Figure 6. Current Pulses with Unstable Operation

Transient Response.

Comparison of the transient speed response times for step changes in firing angle given in Table II for the motor and analogue indicate that the analogue has the same general dynamic characteristic as the actual motor. However, it is obvious that any simulated conditions involving operation in or through regions where the steady-state characteristics of the two differ will not yield reliable results.

As the full connection was made, the oscillations in the circuit were sustained at a value slightly above zero. As a slight delay load, the performance curve, as indicated in Figure 6 (d), again showed a peak of the transient as shown by Figure 6 (b); but this condition resulted in no other action or oscillation from a peak of the transient as shown by Figure 6 (c). The oscillations ceased after the initial increase in loading, oscillations ceased after the initial increase in loading, oscillations ceased after the initial increase in loading.

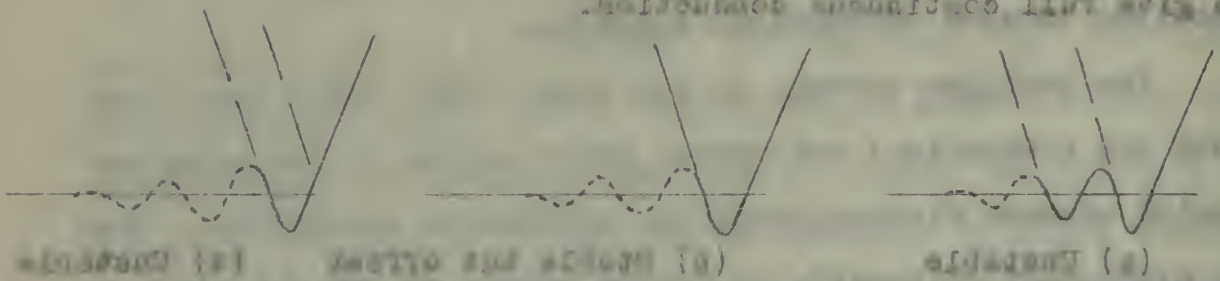


Figure 3. Current levels with variable frequency

• Chlorophyll and carotenoids

Comparison of the treatment of the two cases shows that the results are in good agreement with those obtained in the case of the two cases.

Elimination of Current Cutoff Transient.

From the previous discussion, it is apparent that the major difficulty in the operation of the analogue circuit was the current pulse cutoff transient during discontinuous conduction. From an analysis of the circuit it is evident that when the current decreases to zero and conduction through the rectifier ceases almost instantaneously, the energy stored in the inductance unit is released as a voltage impulse. Since the non-conducting rectifier acts as an open switch, current will flow in the circuit as seen by the inductance only if some path for it exists around the rectifier. Such a path could be provided by the following factors:

- a) stray wiring capacitance in the circuit,
- b) interelectrode capacitance in the thyratrons,
- c) capacitance between the anode transformer secondary windings and ground.

Since all auxilliary equipment such as the phase shift transformer and the thyatron heater transformers were connected to the grounded cathode any capacitance to ground of these units would not be effective.

Removing two of the three thyratrons from the circuit to reduce the effective interelectrode capacitance produced no observable affect on the transient. Mercury vapor type tubes (FG-17) were also substituted with no improvement. By breaking the connection between cathode and ground, however, the amplitude of the transient was reduced appreciably, indicating that the major effects were caused by capacitance

ALL INFORMATION CONTAINED HEREIN IS UNCLASSIFIED

From the above discussion, it is evident that the major difficulty in the operation of the proposed system was the current pulse output. It is suggested that the connection from the terminals of the circuit be made such that when the current decreases to zero and oscillates through the resistor across which inductance is placed, the energy stored in the inductance will be released as a voltage impulse. Since the pre-conditioned resistor does not have a constant value it is difficult to see how the inductance only it does help for it exists around the resistor. Such a unit could be provided by the following system:

- 1) Interference between the two elements usually
occurs in the form of a doublet.

Since all military equipment used in the Cuban conflict
transported and the physical border restrictions were dis-
rupted to the extent between any responsibility to prevent or
allow units would not be considered.

indicating that the above were indeed of significant importance. The results of the experiment were indeed significant, indicating that the above were indeed of significant importance.

to ground of some point between the inductance and the plates of the thyratrons around the lower portion of the circuit as shown in Figure 1. Since the anode transformer cases were not grounded, the most probable cause of the trouble was the interwinding capacitance of these transformers; inasmuch as the primaries provided a path to ground through the power system. The actual capacitance in the transformers was probably of the same order of magnitude as that in the transformers used with the ignitron rectifier; but because of the scaled down parameters of the analogue circuit, its effect was magnified a thousand times.

Smaller heater type transformers were tried in place of the large anode transformers, but no appreciable difference was noted. These and the large transformers were connected in zig-zag instead of delta-wye, also causing no improvement.

It was found that with a 50,000 ohm resistor in the ground connection the transient was damped out in the minimum time, although its peak amplitude was somewhat greater than with the ground circuit open. This arrangement was used for all the tests.

After the analogue circuit was disassembled, it was found that on the actual ignitron rectifier a 2000 ohm resistor was permanently installed directly across the line connected to the d-c output. A corresponding resistor was not tried in the analogue circuit, but it is probable that it would have about the same effect as the resistor in the ground connection.

to ground of each point between the insulation and the plane of the electrodes around the lower portion of the circuit as shown in Figure 1. When the mode of operation was changed and the electrodes, the two parallel plates of the condenser were insulated, the resistance of these electrodes; however, the electrodes provided a path to ground through the lower system. The actual resistance in the electrodes was only of the same order of magnitude as that in the system. The electrodes used were the same material; the thickness of the metal was measured at the electrodes and the effect was negligible - a known value.

Smaller capacitor type electrodes were used in place of the large metal electrodes, but no appreciable difference was noted. These and two large electrodes were connected in a way similar to that shown, also showing no improvement.

It was found that with a 50,000 ohm resistor in the ground connection the resistance was changed and in the same time, although the same resistance was measured between them with the same circuit. This arrangement was used for all the tests.

After the electrodes were disconnected, it was found that on the actual circuit specified a 5000 ohm resistor was permanently installed across the two electrodes to the ground. A potentiometer circuit was used in the circuit, but it is possible that it would have about the same effect as the resistor in the ground connection.

Determination of the Equivalent Armature Resistance.

One of the problems involved in setting up the analogue circuit is the determination of the equivalent armature resistance of the motor. The resistance unit used in the analogue circuit had the same d-c and a-c resistance up to 1000 cps. However, the measured 60 cycle a-c resistance of the actual motor was more than twice the measured d-c resistance. The 60 cycle a-c resistance of the motor was used in determining the equivalent for the analogue circuit; and although no justification for this can be found, the correspondence of the actual performance curves with the theoretical and analogue curves indicates that this gives better results than if the d-c resistance had been used.

EVALUATION OF THE ANALOGUE CIRCUIT

The evaluation of the analogue circuit may be broken down into two aspects according to the use for which it is intended; first for obtaining the steady-state speed (counter-emf) versus armature current characteristics, and second, for transient investigations involving the use of feedback.

Steady-State.

Even though the basic steady-state phenomena of the system are properly represented in the analogue circuit, the performance curves are not accurate in the boundary region between discontinuous and continuous conduction as explained in the previous section. Since these inaccuracies cover a considerable portion of the overall curves, and since it does not appear possible to completely eliminate the current oscillations which cause them, the analogue circuit is not considered practicable for obtaining complete accurate steady-state performance data.

Transient.

The analogue circuit may be useful, however, for transient investigations not requiring precise correspondence in the regions where the characteristics are inaccurate or the operation unstable. The important area for transient investigation using feedback appears to be the problem of improving the slowdown characteristics. Another field for investigation is the stabilization of speed with change in load. It is considered practicable to use the analogue

STABILITY OF THE SYSTEM

The variation of the modulus of Δ with ω is shown in Fig. 2. The two curves correspond to the case when Δ is calculated; first for the case of the steady-state speed (solid line) and second (dashed line) for the case of the transient speed (dashed line). The curves show that the system is stable for all values of ω .

Even though the basic theory-state phenomenon of the system are relatively independent in the steady state, the performance curves are not identical in the transient region between the two cases. This is explained in the previous section. Since these phenomena cover a considerable portion of the overall drive, and since it does not appear possible to completely eliminate the current oscillations which occur here, the analysis already is not considered precise for predicting complete motor steady-state performance data.

Transient

The analysis already may be useful, however, for transient investigations not requiring precise quantitative data in the region where the characteristics are insensitive to the operating method. The transient time for transient investigation using feedback appears to be the problem of improving the system characteristics. Another field for investigation is the stabilization of speed with regard to load. It is considered possible to use the analysis

circuit for investigations along either of these lines. However, any feedback work involving stability will be complicated by the inherent instability of the analogue circuit in certain regions.

However, our findings were interpreted differently with respect to the importance of the findings in certain regions.

ANALYTICAL PREDICTION OF TRANSIENT RESPONSE

An analysis of the equivalent circuit of the d-c machine and its load together with performance curves for the ignitron-motor combination leads to a relatively simple means of predicting the transient response of the system for changes in firing angle or load without resorting to actual experiment.

Equivalent Circuit.

As previously discussed, a separately excited d-c machine and its load may be represented by the equivalent circuit of Figure 7.

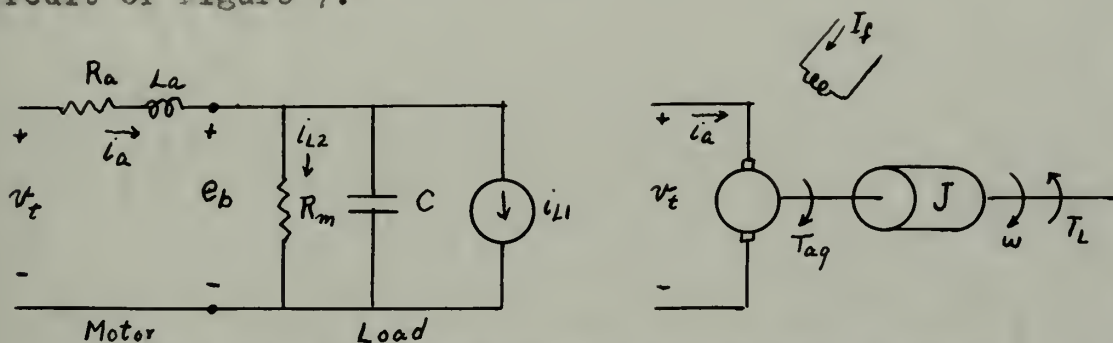


Figure 7. D-C Motor and Equivalent Circuit.

$$\text{For } I_f \text{ constant} \quad T_{a9} = K i_a \quad (9)$$

$$J = K^2 C \quad (10)$$

$$E_b = K \omega \quad (11)$$

$$T_{L1} = K i_{L1} \quad (12)$$

$$T_{L2} = K i_{L2} = \frac{E_b}{K R_m} = \frac{K^2 \omega}{R_m} \quad (13)$$

For a linear analysis i_{L1} and R_m are constant if the machine is running, simulating a load with a component of constant torque and a component that is directly proportional to speed. Using mks units $k_1 = k_2 = K$.

ANALYSIS OF THE SYSTEM WITH A LOAD

An analysis of the system with a load is shown in Figure 7. The load is represented by a resistor R_L in series with a capacitor C . The load is connected to the system at the output of the motor. The load is represented by a resistor R_L in series with a capacitor C . The load is connected to the system at the output of the motor.

Equivalent Circuit

As previously discussed, a separate model for the machine and its load may be represented by the equivalent circuit of Figure 7.

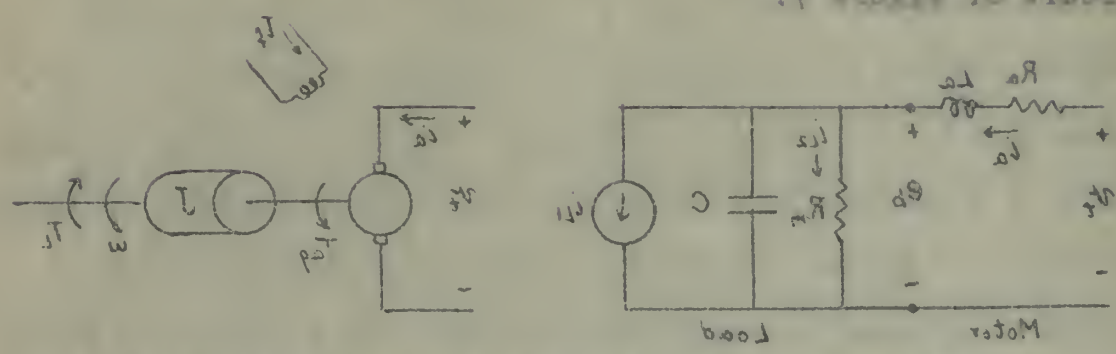


Figure 7. D-C motor and equivalent circuit.

- (1) For I_a constant $T_{ad} = T_{ad}$
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Steady-State Analysis.

The steady-state speed-torque curves for such a motor with various values of terminal voltage are as shown in Figure 8, and are derived from the relationship

$$E_b = V_t - i_a R_a \quad (14)$$

$$\text{or} \quad w = \frac{V_t}{K} - \frac{R_a}{K^2} T_{ag} \quad (15)$$

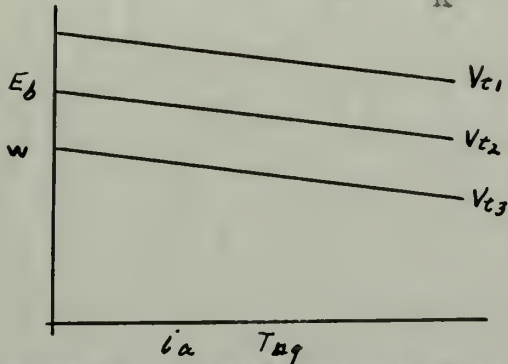


Figure 8. Speed-Torque Characteristics for Separately Excited D-C Motor.

The slope of these lines is

$$\frac{\delta E_b}{\delta i_a} = -R_a \quad (16)$$

$$\text{or} \quad \frac{\delta w}{\delta T_m} = \frac{R_a}{-K^2} \quad (17)$$

If a particular load characteristic is plotted on the same coordinates, it is a straight line as shown in Figure 9 with the horizontal axis intercept of i_{L1} or T_{L1} and slopes

$$\frac{\delta E_b}{\delta i_L} = R_m \quad (18)$$

$$\frac{\delta w}{\delta T_L} = \frac{R_m}{K^2} \quad (19)$$

Figure 8. Speed-Torque Characteristics for

The speed-torque characteristics for the motor

with various values of terminal voltage are shown in

Figure 8, and the derivation from the relationship

$$V_t = I_a R_a + E_b \quad (11)$$

$$E_b = \frac{V_t}{k} - \frac{I_a R_a}{k} \quad (12)$$

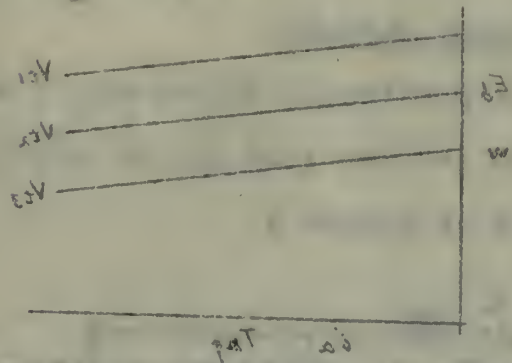


Figure 8. Speed-Torque Characteristics for
Constantly Excited D-C Motor.

The slope of these lines is

$$\frac{\partial E_b}{\partial I_a} = -R_a \quad (13)$$

$$\text{or} \quad \frac{\partial n}{\partial I_a} = \frac{R_a}{k} \quad (14)$$

If a particular load characteristic is plotted on the
same coordinates, it is a straight line as shown in Figure 9
with the horizontal axis intercept of I_{a0} on I_a and passes

$$\frac{\partial E_b}{\partial I_a} = R_a \quad (15)$$

$$\frac{\partial n}{\partial I_a} = \frac{R_a}{k} \quad (16)$$

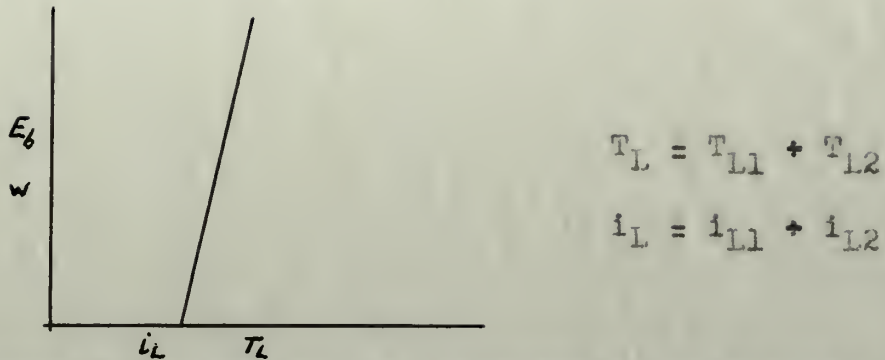


Figure 9. Speed-torque Characteristics of Load.

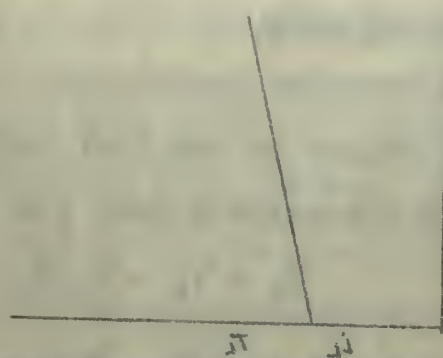
The steady-state operating conditions for a particular load and terminal voltage are given by the intersection of the load and motor speed-torque curves.

This same reasoning may be applied to the ignitron-motor combination to determine the steady-state operating conditions. In this case, however, the motor speed-torque curves are not straight lines but are similar to Figure 2, and are plotted for firing angle as a parameter rather than applied terminal voltage.

Transient Analysis.

The transient of primary interest is the behavior of motor speed for a change in firing angle. The transient analysis is most easily performed by considering the electrical equivalent circuit of Figure 7 rather than the mechanical system. The electro-mechanical conversion factor, K , is a constant if armature reaction is neglected or assumed constant, permitting a conversion at any stage of the analysis from electrical to mechanical parameters.

The armature inductance will be neglected since, for most integral horsepower motors used in power applications,



[Faint mirrored bleed-through from reverse side]

The steady-state operating conditions for a particular load and terminal voltage are given by the intersection of the load and motor speed-torque curves.

This same reasoning may be applied to the induction motor operation at determining the steady-state operating conditions. In this case, however, the motor speed-torque curves are not straight lines but are plotted for firing angle as a parameter. Figure 2, applied terminal voltage.

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The treatment of primary interest is not however of
major interest for a change in living habits. The treatment
analysis is not easily left out of consideration and the
clinical evaluation of the effect of the treatment is not
an ideal system. The direct-reading system is not ideal, it
is a constant if it is not treated in a constant or constant
constant, providing a constant in the state of the analysis
the treatment is mechanical treatment.

its effect is negligible in comparison with the effect of the motor and load inertia or its equivalent capacitance. Furthermore, under conditions of discontinuous conduction through the ignitron rectifier, there is no current flowing in the armature circuit during a portion of each cycle and no net energy is stored in the inductance from cycle to cycle; consequently the speed transient is not affected by the inductance under these circumstances. With continuous conduction, the inductance may be effective; but, as will be brought out subsequently, the transient performance of the motor under these conditions is not affected by the rectifier and presents no new problems.

With the armature inductance omitted, the circuit of Figure 7 may be converted into an equivalent as shown in Figure 10.

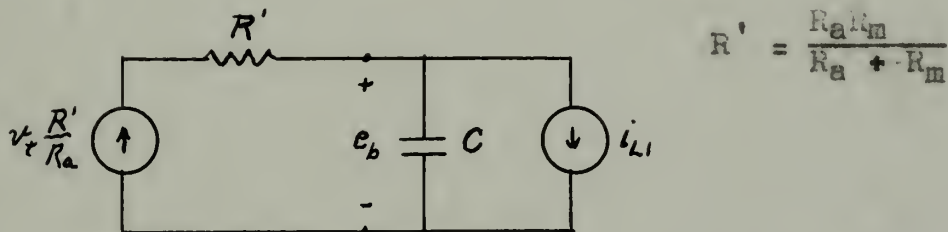


Figure 10. Equivalent Circuit for D-C Motor and Load.

The Kirchhoff equations for this circuit are

$$v_t \frac{R'}{R_a} = i_a' R' + e_b \quad (20)$$

$$i_a' = i_{L1} + C \frac{de_b}{dt} \quad (21)$$

$$v_t \frac{R'}{R_a} - R' i_{L1} = R' C \frac{de_b}{dt} + e_b \quad (22)$$

the effect is negligible in comparison with the effect of the motor and load in the equivalent circuit. Furthermore, under conditions of maximum induction between the field and rotor, there is no current flowing in the armature circuit during a portion of each cycle and no energy is stored in the inductance from cycle to cycle; consequently the speed variation is not affected by the inductance under these circumstances. All conditions considered, the inductance may be neglected; but, it will be brought out subsequently, the transient performance of the motor under these conditions is not affected by the inductor and presents no new problems.

With the armature inductance neglected, the circuit of Figure 2 may be represented into an equivalent circuit as shown in Figure 3.

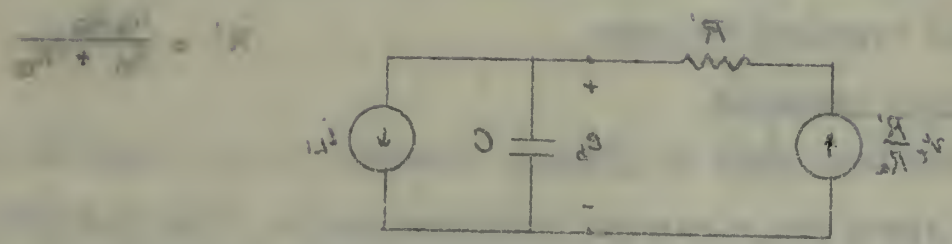


Figure 3. Equivalent circuit for motor and load.

The following equations for this circuit are:

$$V = I' R' + \frac{1}{C} \int I' dt \quad (50)$$

$$I' = \frac{1}{R'} \left(V - \frac{1}{C} \int I' dt \right) \quad (51)$$

$$\frac{dI'}{dt} + \frac{1}{R' C} \int I' dt = \frac{V}{R'} \quad (52)$$

For a step change in either v_t or i_L

$$e_b = E_{bi} + (E_{bf} - E_{bi}) \left(1 - e^{-\frac{t}{R'C}} \right) \quad (23)$$

where E_{bi} = initial E_b corresponding to initial speed

E_{bf} = final E_b corresponding to final steady-state speed

To apply this result to the ignitron-motor combination it is necessary to determine the values of R_a , R_m , E_{bi} and E_{bf} to be used. These will be arrived at by investigating the actual behavior of the circuit or motor during the transient.

Considering first the performance of the d-c motor with speed-torque characteristics as in Figure 8 for a step change in terminal voltage, the sequence of events is as shown in Figure 11 as follows.

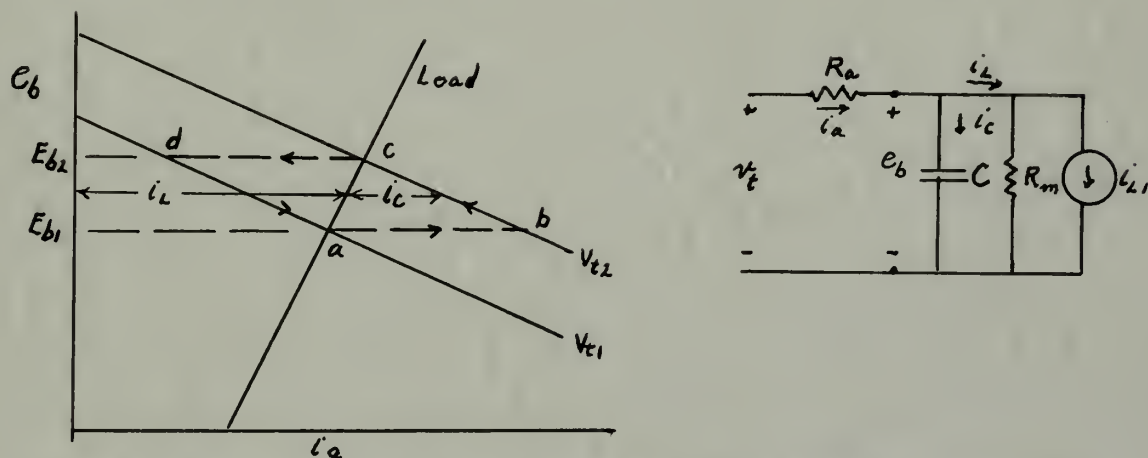


Figure 11. D-C Motor Performance for Speed Transient.

The initial conditions for the given load and terminal voltage V_{t1} are given by the steady-state operating point at a with $e_b = E_{b1}$ and $i_a = i_L$.

For a step change in speed ω of the motor, the transfer function is given by

$$\omega(s) = \frac{1}{s} \left[\frac{1}{s} + \frac{1}{s} \left(\frac{1}{\tau_m} + \frac{1}{\tau_e} \right) + \frac{1}{s} \right] \quad (23)$$

where τ_m = motor time constant, τ_e = field time constant.

To apply this result to the speed-torque characteristic it is necessary to determine the values of τ_m and τ_e for the motor. These will be obtained by investigating the actual behavior of the motor at various speeds.

Consider first the performance of the motor with speed-torque characteristics as in Figure 8 for a step change in terminal voltage. The response of speed is as shown in Figure 11 as follows.

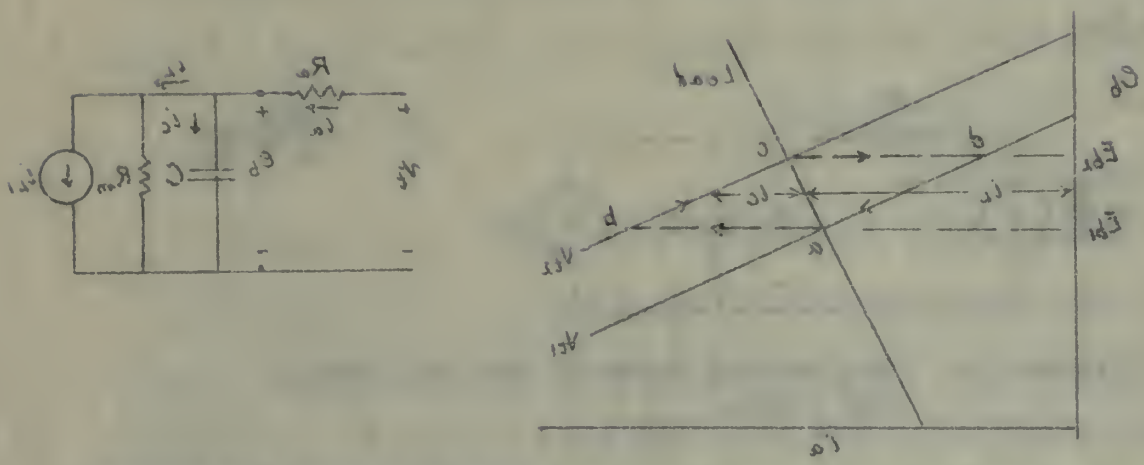


Figure 11. Speed-torque characteristics for a step change in terminal voltage.

The initial conditions for the motor are assumed to be given by the steady-state operating point at $\omega = \omega_a$ and $T = T_a$.

At $t = 0$ the terminal voltage is changed to V_{t2} , and at $t = 0+$ the operating point has moved horizontally to point b, since the charge on the condenser and hence e_b cannot change instantaneously. For $t > 0$ the charging current to the condenser ($i_c = i_a - i_L$) is given by the horizontal distance between the motor curve and load curve; and as the condenser charges, e_b increases so that the motor operating points move along the line b-c until the final steady-state condition is reached at point c.

Similarly, if the initial conditions are taken at point c, and the terminal voltage is reduced from V_{t2} to V_{t1} , the operating point moves to point d at $t = 0+$. In this case for $t > 0$, i_c is negative since i_a is less than i_L , and the charge on the condenser is reduced, decreasing e_b along the curve d-a until the final steady condition at point a is reached.

This reasoning may be clearer if applied to the mechanical operation of the motor. If a step change is made in the terminal voltage, the speed and hence the back-emf cannot change instantaneously because of the load (or rotor) inertia. The armature current increases almost instantaneously, however, to $\frac{V_{t2} - E_{b1}}{R_a}$ and produces an air gap torque T_{ag} which provides the load torque, T_L , and an accelerating torque, T_a . As the rotor accelerates, the speed and back-emf increase and i_a and T_{ag} decrease until the new steady state condition is reached. The detailed analysis of the electrical

equivalent may be applied to the mechanical case by taking $T_m = K i_a$, $T_L = K i_c$ and the speed equal to e_b/K .

For the non-linear performance curves of the ignitron-motor combination a similar analysis may be carried out for a change in firing angle as shown in Figure 12.

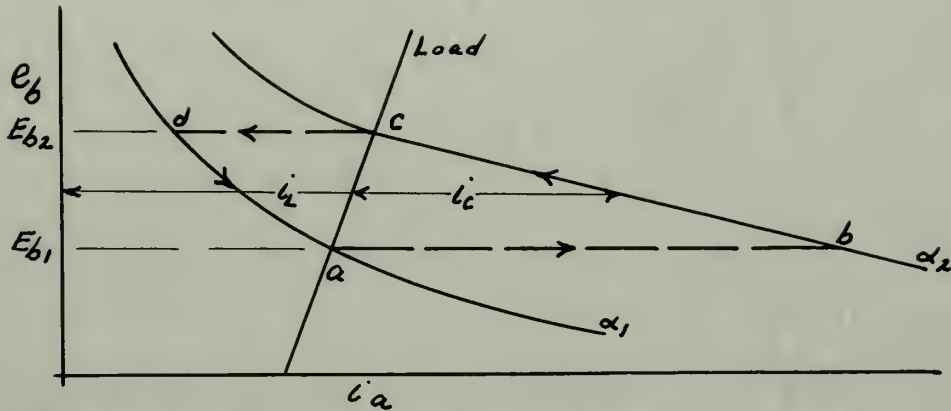


Figure 12. Ignitron-motor System Performance for Speed Transient.

For a step change in firing angle from α_1 to α_2 the output shifts from point a to point b at $t = 0+$ and for $t > 0$ moves along the α_2 curve from b to c until the final steady-state condition at c is reached. Similarly for a change from α_2 to α_1 , the output moves from point c to point d at $t = 0+$ and then moves along the α_1 curve until the steady-state condition at a is reached.

To determine the appropriate values of E_{b1} , E_{b2} , R_a and R_m for the ignitron-motor system necessary for obtaining the analytical expression for the transient performance of e_b given by equation (23), it is only necessary to compare the linear d-c motor and non-linear ignitron motor curves of Figures 11 and 12. In Figure 11, R_a is the nega-

equivalent may be applied to the mechanical case of linear
 $\dot{m} = \dot{m}_0$, $\dot{V} = \dot{V}_0$ and $\dot{Q} = \dot{Q}_0$.
 For the non-linear perturbation curves of the linear-
 error considered a similar analysis may be carried out for
 a change in the α value as shown in Figure 15.

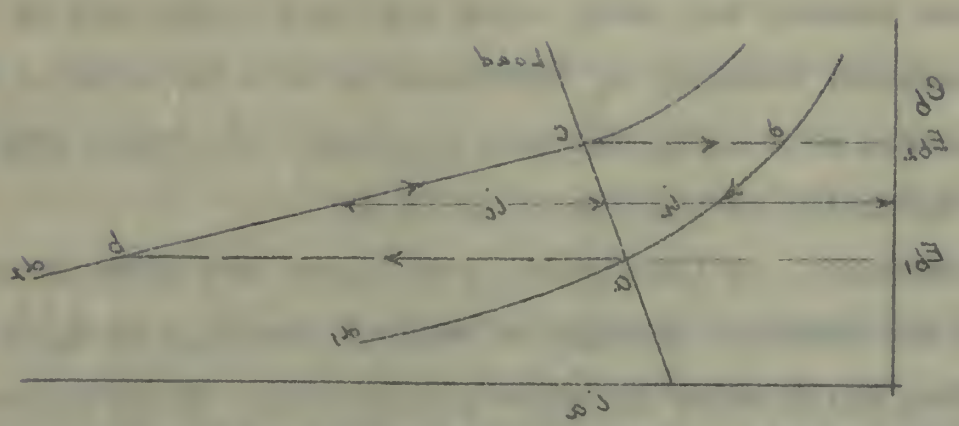


Figure 15. Linear-error system perturbation
 for speed disturbance.

For a step change in the α value from α_1 to α_2 the
 output will move from point a to point b at $t = 0$ and for
 $t > 0$ moves along the α_2 curve from b to c until the \dot{m}
 steady-state condition is reached. Similarly, for a
 change from α_2 to α_1 , the output moves from point b to
 point a at $t = 0$ and then moves along the α_1 curve until
 the steady-state condition is reached.
 To determine the approximate values of \dot{m} , \dot{V} , \dot{Q} , \dot{P}
 and \dot{E} , for the linear-error system, the same procedure is used
 for the analytical, numerical, or graphical determination
 of α as given by equation (11). It is only necessary to use
 the linear α -value and non-linear \dot{m} value for
 curves of Figures 11 and 12. In Figure 11, \dot{m} is the new-

tive slope of the V_{t1} and V_{t2} curves along which the motor output moves during the transient. By analogy, R_a in Figure 12 is the negative slope of the curves c-b and d-a and is different for the conditions of increasing and decreasing speed. In both cases, however, R_m is the slope of the load characteristic and E_{b1} and E_{bf} are the values of e_b at $t = 0+$ (point b or d) and $t = \infty$ (point e or a).

Summary.

The speed transients for the ignitron-motor system for a step change in firing angle can thus be predicted by use of the electrical performance curves for the combination.

The procedure is summarized as follows:

- a) Plot on the ignitron-motor curves the electrical equivalent of the load speed-torque curve. This curve need not be a straight line.
- b) For the initial and final firing angle α_i and α_f , read off E_{b1} and E_{bf} .
- c) Determine R_a from the average slope of the α_f curve between the points where $e_b = E_{b1}$ and $e_b = E_{bf}$.

$$(R_a = -\frac{\delta e_b}{\delta i_a}).$$

- d) Determine R_m from the average slope of the load curve between the points where $e_b = E_{b1}$ and $e_b = E_{bf}$.

$$(R_m = \frac{\delta e_b}{\delta i_L}).$$

- e) Knowing the electrical equivalent of the inertia

$C = \frac{J}{K^2}$, find the time constant for this condition.

$$\tau = \frac{R_a R_m}{R_a + R_m} \times C \quad (24)$$

From slope of the \log and the number of points along the curve
 during the initial and final time intervals, α_i and α_f ,
 Figure 1 is the logarithmic slope of the curve at the initial
 and final time intervals. The curve is assumed to be continuous
 and smooth. In the case of a constant slope, $\alpha_i = \alpha_f = \alpha$,
 the curve is a straight line. The slope of the curve at the
 initial and final time intervals is α_i and α_f respectively.
 $\alpha = 0$ (initial α of 0) and $\alpha = \infty$ (final α of ∞).

The general procedure for the logarithmic-slope system for
 a step change in liquid level has been described in the
 of the electrical resistance curve for the system.
 The procedure is summarized as follows:

- a) Plot the logarithmic-slope curve and the electrical
 resistance of the liquid level curve. This
 curve may be a straight line.
- b) For the initial and final time intervals α_i and α_f ,
 find the \log and \log .
- c) Determine α from the average slope of the \log α .
 The average between the points where $\alpha_i = \alpha_f = \alpha$.

$$\log \frac{\delta u}{\delta t} = \frac{\delta u}{\delta t}$$

- d) Determine α from the average slope of the \log α .
 The average between the points where $\alpha_i = \alpha_f = \alpha$.
- e) $\log \frac{\delta u}{\delta t} = \frac{\delta u}{\delta t}$

f) Knowing the electrical resistance of the liquid
 level, find the time constant for the system.

$$\tau = \frac{R}{\alpha} \quad (34)$$

f) The transient for E_b is then given by

$$K_w = e_b = E_{bi} + (E_{bf} - E_{bi}) (1 - e^{-t/\tau}) \quad (25)$$

The speed transient for a step change in load can be predicted using the above procedure modified in the following respects:

- a) Initial and final load speed-torque curves are plotted.
- b) E_{bi} and E_{bf} are found from the two load curves and the given firing angle curve.
- c) R_m is determined from the slope of the final load curve.

Special Conditions.

For certain situations using the above procedures, the slope along either curve may change radically during the course of the transient as illustrated in Figure 13. In these instances the speed transient may be broken down into two or more successive portions, each portion characterized by a time constant (equation (24)) obtained from the slopes of straight line approximations to the motor and load curves, and with E_{bf} determined from the intersection of the two assumed straight lines starting at E_{bi} on the two curves. When e_b reaches a value where the curve slopes change appreciably for the succeeding portion of the transient, new straight line approximations are made for determining a new time constant and E_{bf} as before, but for this portion E_{bi} is taken as the value of e_b at which the first portion of the transient was terminated.

7) The treatment for β is then given by
$$\beta = \beta_1 + (\beta_2 - \beta_1) \left(\frac{1 - \beta}{1 - \beta_1} \right)^{1/\beta_2}$$

The speed treatment for a step change in load can be predicted using the above procedure modified to the following respect:

a) Initial and final load speed curves are plotted.

b) β_1 and β_2 are found from the two load curves and the given timing data curve.

c) β is determined from the slope of the final load curve.

Special Conditions

For certain situations among the above procedures, the slope along either curve may change radically during the course of the treatment as illustrated in Figure 2). In these instances the speed treatment may be broken down into two or more successive portions, each portion characterized by a time constant (assumed β_1) obtained from the slopes of straight line approximations to the load and speed curves, and with β_2 determined from the intersection of the two assumed straight lines occurring at β_1 on the two curves. Then at testing a value across the curve slopes changes slightly for the succeeding portion of the treatment, and straight line approximations are made for determining a new time constant and β_2 as before, and for this testing β_1 is taken as the value of β_2 at which the first portion of the treatment was terminated.

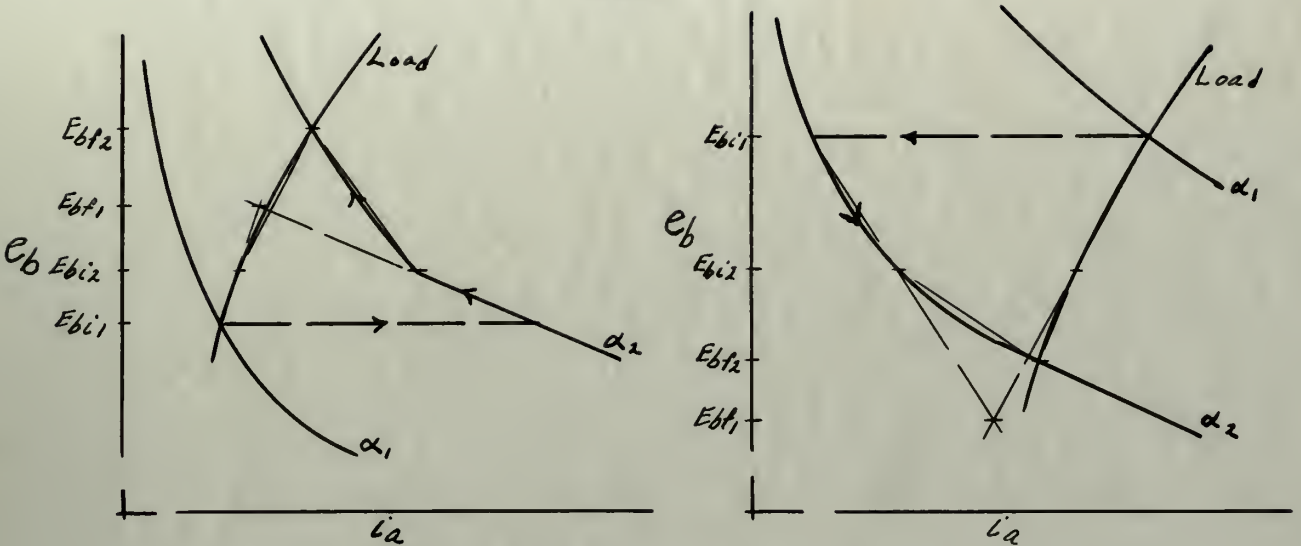


Figure 13. Transients With Change of Slope.

The speed transient for either condition of Figure 13 and a step change in firing angle from α_1 to α_2 is given by

$$e_b = E_{bi1} + (E_{bf1} - E_{bi1}) (1 - e^{-t/\tau_1}) \quad (25a)$$

$$\text{for } 0 < t < t_1$$

$$\tau_1 = \frac{R_{a1} R_{m1}}{R_{a1} + R_{m1}} \times C \quad (24a)$$

$$e_b = E_{bi2} + (E_{bf2} - E_{bi2}) (1 - e^{-t/\tau_2}) \quad (25b)$$

$$\text{for } t > t_1$$

$$(24b)$$

$$\tau_2 = \frac{R_{a2} R_{m2}}{R_{a2} + R_{m2}} \times C$$

where t_1 is the value of t for which e_b of equation (25a) equals E_{bi2} .

This same line of reasoning may be applied in finding the transient speed response to any arbitrary input by a step by step method or even to a sinusoidal input if the operating output is plotted point by point on the perform-

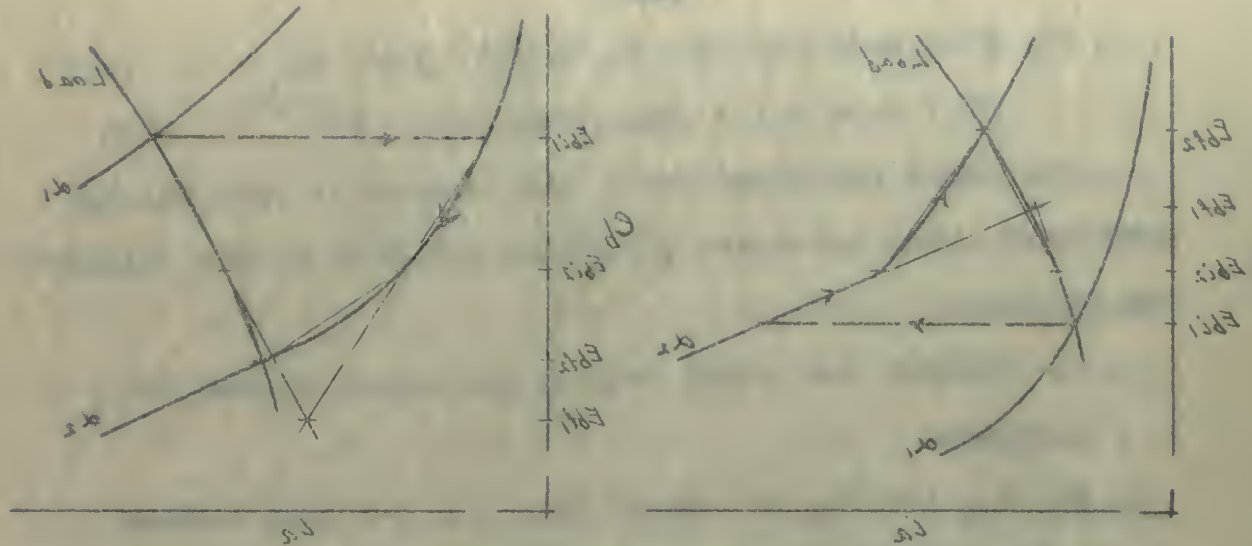


Figure 13. Transient with Change of Slope.

The speed transient for either condition of Figure 13 and a step change in firing angle from α_1 to α_2 is given by

$$0 = \sin(\alpha_1 + \cos(\alpha_1 - \alpha_2)) (1 - e^{-\tau/\tau_1})$$

for $\tau > \tau_1$

$$\tau_1 = \frac{\sin(\alpha_1 + \cos(\alpha_1 - \alpha_2))}{\sin(\alpha_1 - \alpha_2)}$$

$$0 = \sin(\alpha_2 + \cos(\alpha_2 - \alpha_1)) (1 - e^{-\tau/\tau_2})$$

for $\tau > \tau_2$

$$\tau_2 = \frac{\sin(\alpha_2 + \cos(\alpha_2 - \alpha_1))}{\sin(\alpha_2 - \alpha_1)}$$

where τ_1 is the value of τ for which α of equation (13a) equals α_1 .

This same line of reasoning may be applied to finding the transient speed response to an arbitrary load by an step by step method or even to a sinusoidal input if the operating output is plotted point by point as was before.

ance curves and the proper values are used for the time constant and forcing function. In general, however, the three-phase ignitron-motor performance for large disturbances is very non-linear, and no simple transfer function relating dynamic output to input can be derived for it.

Current Transients.

If an analysis for the average armature current transient is carried out similar to the above presentation, it will be found that the equation is similar in form to equation (25) and is characterized by the same time constant, equation (24). The current transient is given by

$$\begin{array}{l} i_a = I_{ai} + (I_{af} - I_{ai}) (1 - e^{-t/\tau}) \\ \text{for} \\ t > 0 \end{array} \quad (26)$$

where $I_{ai} = i_a$ at $t = 0+$

All subscripts have the same meaning as in equations (24) and (25) as indicated on Figure 12 for the voltage transient, and the value of τ is determined in the same manner.

These observations and the present results are used for the first
coherent and logical analysis. In general, however, the
three-phase system model is not applicable for large systems
where it is very inefficient, and in some cases it may lead
to misleading results. It must be noted that it

[illegible]

equation (24). The output impedance is given by equation (25) and is characterized by the same time constant, it will be found that the impedance is similar to that of a series inductor in parallel with a resistor. If an analysis for the average structure output is desired

$$(12) \quad (1/2 - \epsilon - 1) (I_2 I - I_1 I) + I_2 I = 0$$

COMPARISON OF EXPERIMENTAL AND PREDICTED SPEED TRANSIENTS

The analytical method for predicting speed transients described in the previous section was used to obtain the theoretical response curves of Figures 14 to 18 for the motor and operating conditions used in Heller's investigation.⁴⁾ These are compared with the experimental response curves which he obtained from oscillograph data, and show very close agreement in nearly all instances.

The speed-torque (counter-emf versus armature current) curves used were those of Figure 2 obtained for the ignitron-motor combination as described on page 49. The actual operating conditions⁴⁾ and the values of R_a derived from the performance curves together with the calculated time constant used for determining the theoretical transient response for the various runs are shown in Table III. The method of plotting the response in two parts with different time constants was used only for runs P-1 and J-2. Additional accuracy could be obtained by dividing some of the other transients up in the same manner, but the results as they stand are considered sufficiently accurate for engineering work.

This method of predicting transients is considered to be more satisfactory than either the full scale tests or tests with an analogue circuit. Furthermore, it points up the limitations of the ignitron-motor combination in terms of the physical parameters of the system. The major difficulty

THEORY OF THE RELATIONSHIP BETWEEN THE TWO

The model is based on the following assumptions:
described in the previous section was used to obtain the
theoretical response curves of the system as a function of the
and operating conditions used in the model's investigation.
These are compared with the experimental response curves
which are obtained from the model's response data, and then very
close agreement is usually obtained.

The model is a computer-aided model (see Appendix A)
curves and were taken of the system obtained for the various
model conditions as described in Table A. The model oper-
ating conditions are the values of the various parameters
for the various curves obtained with the model. The model
used for determining the theoretical response curves for
the various cases are shown in Table A. The model of
obtaining the response in two parts with different time con-
stants was used only for cases 1-1 and 1-2. Additional notes
may be obtained by dividing some of the other parameters
up in the same manner, and the results are then found and
considered individually for each case.
This method of predicting the results is considered to
be more satisfactory than either the full scale tests or
tests with an actual system. Furthermore, it points up
the limitations of the system-response relationship in terms
of the physical properties of the system. The major difficulty

in using this method is the problem of obtaining the basic counter-emf versus armature current performance curves.

These curves may be obtained analytically, however, without actual tests on the system as will be described in the following section.

in which this factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The first factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The second factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The third factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The fourth factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The fifth factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

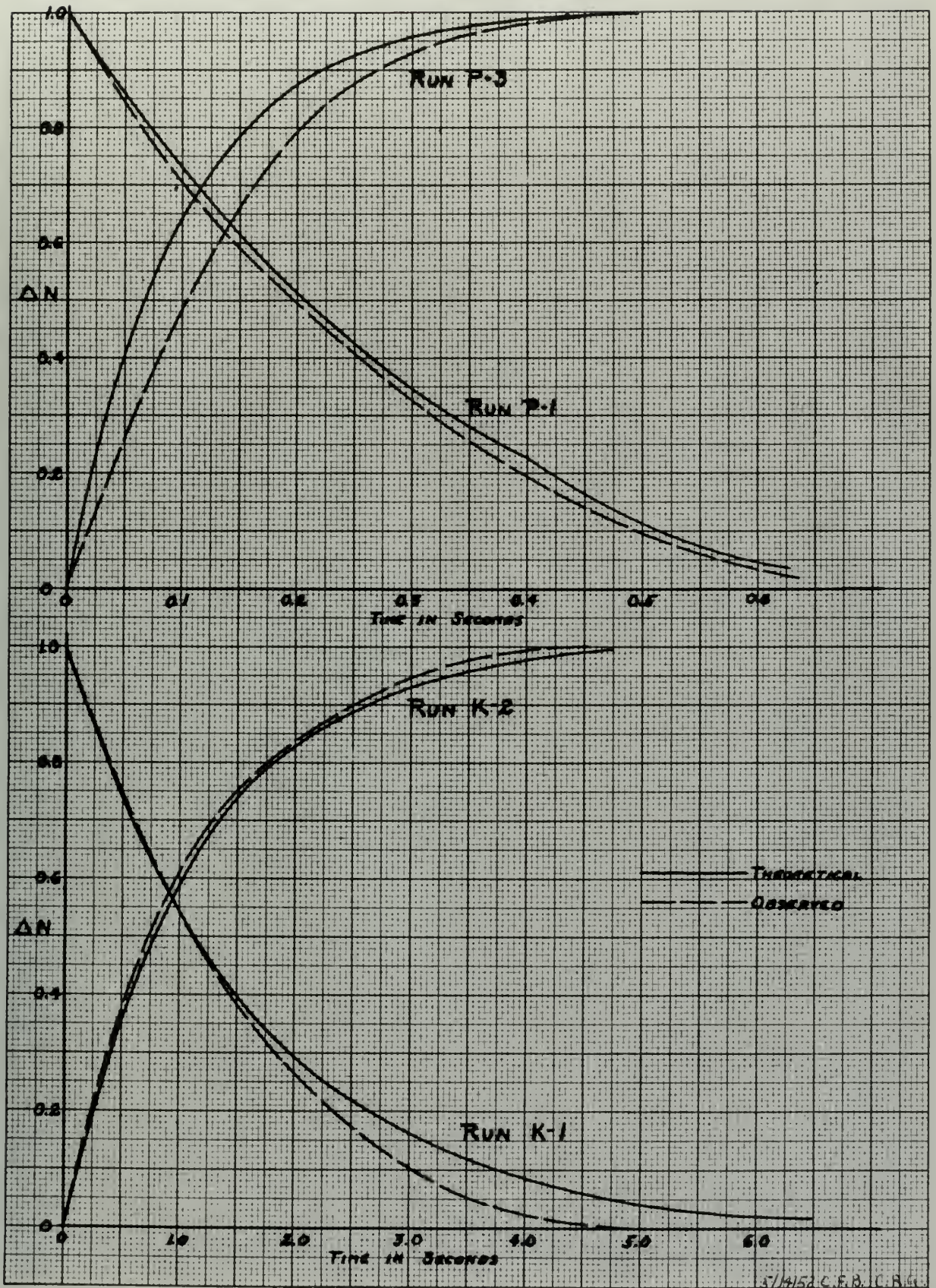
The sixth factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The seventh factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The eighth factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

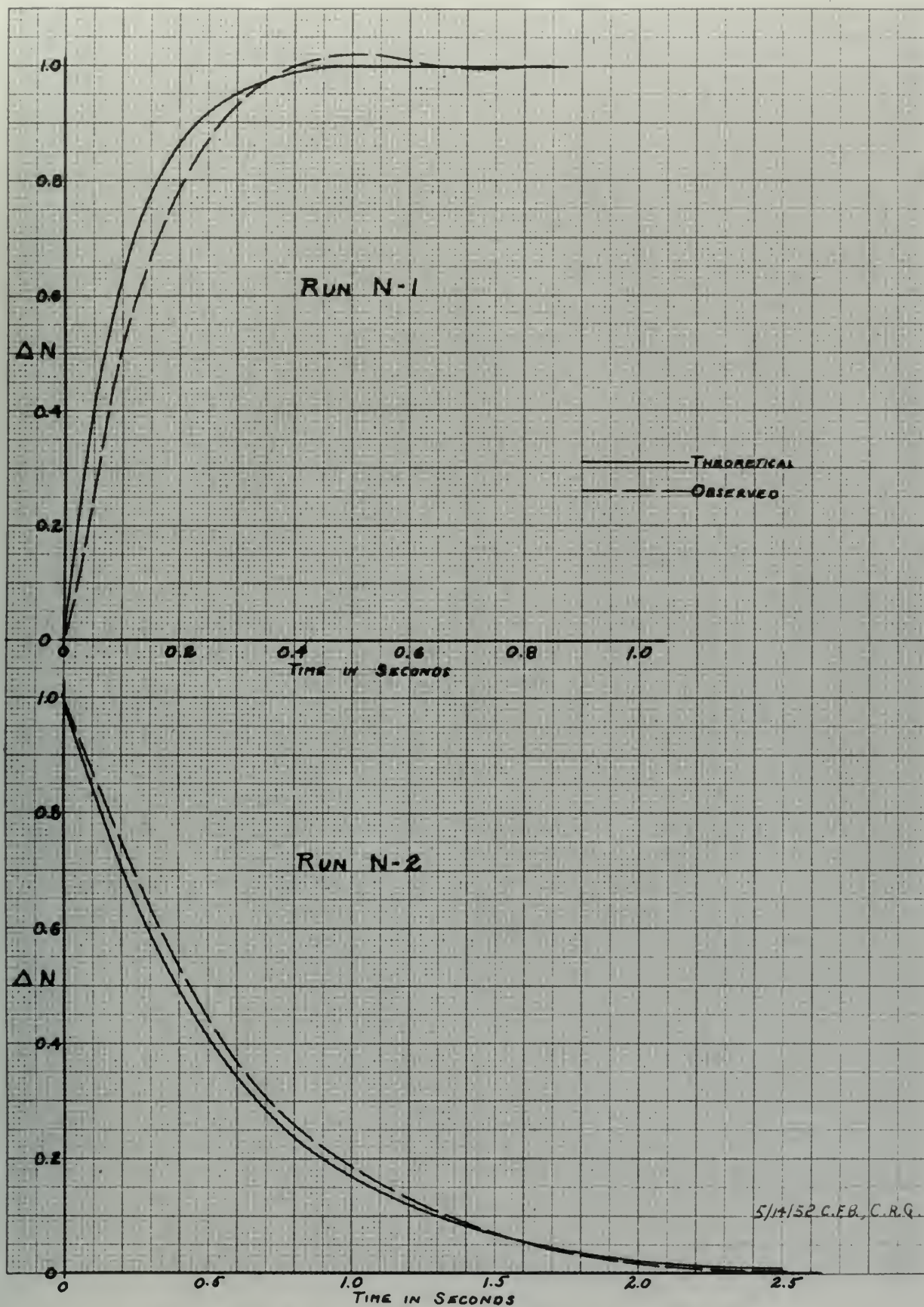
The ninth factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.

The tenth factor is the number of children the child
owned-and various structural factors are involved.
These factors are in general analytically, however, without
actual facts on the subject as will be discussed in the
following section.



SPEED TRANSIENT FOR STEP CHANGE IN FIRING ANGLE
IGNITRON MOTOR CONTROL

FIG 14



SPEED TRANSIENT FOR STEP CHANGE IN FIRING ANGLE
IGNITRON MOTOR CONTROL

FIG 15

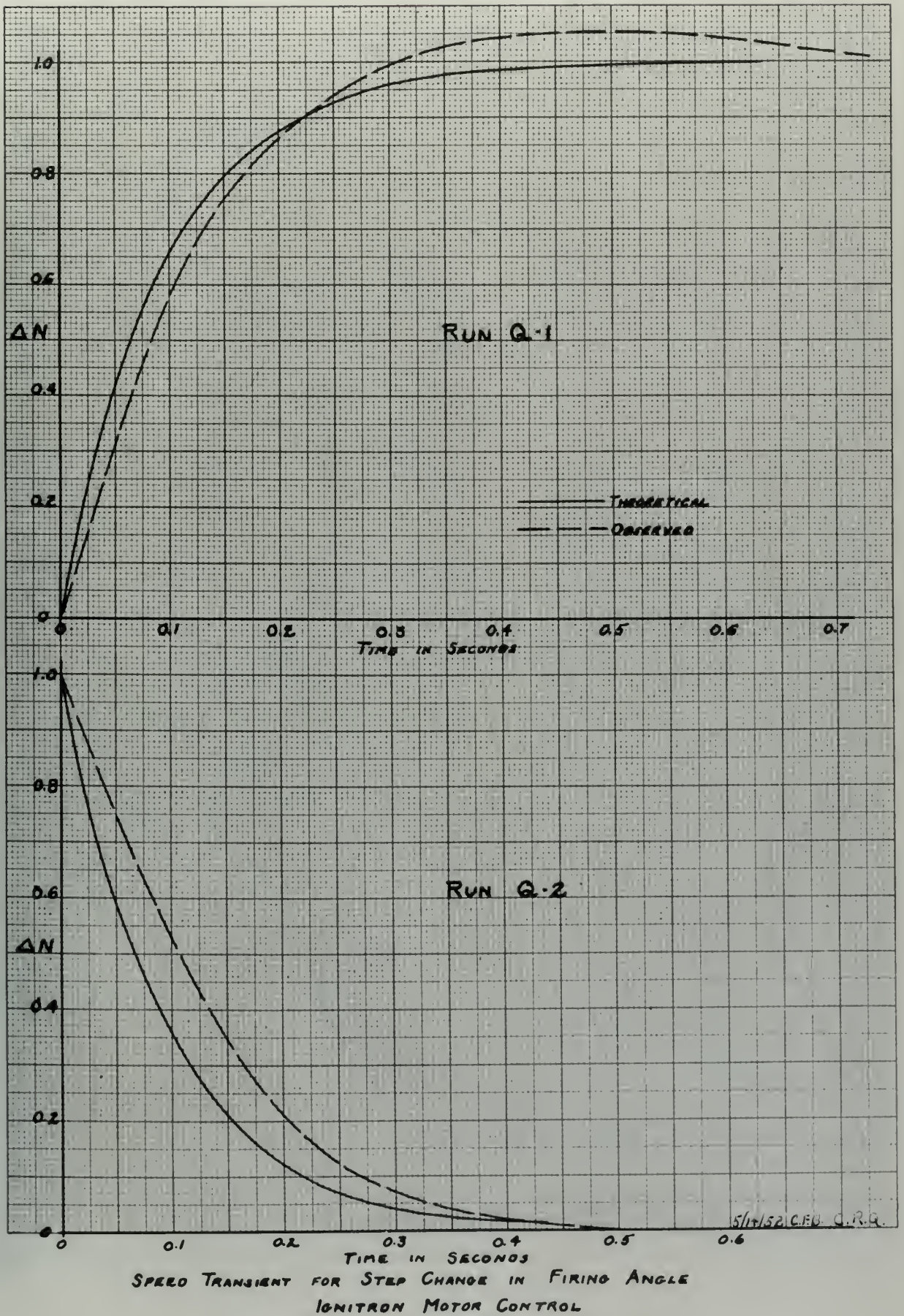
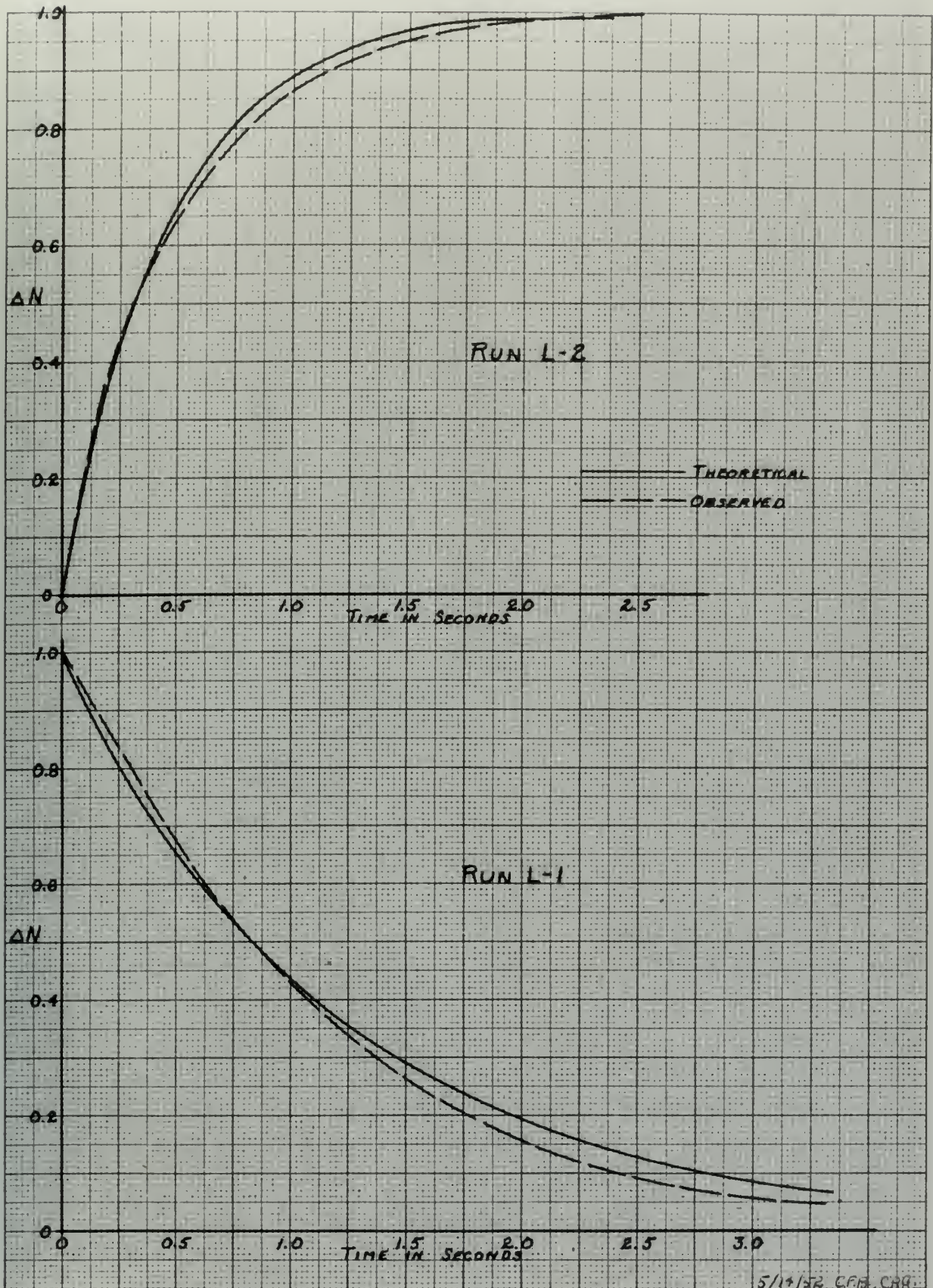
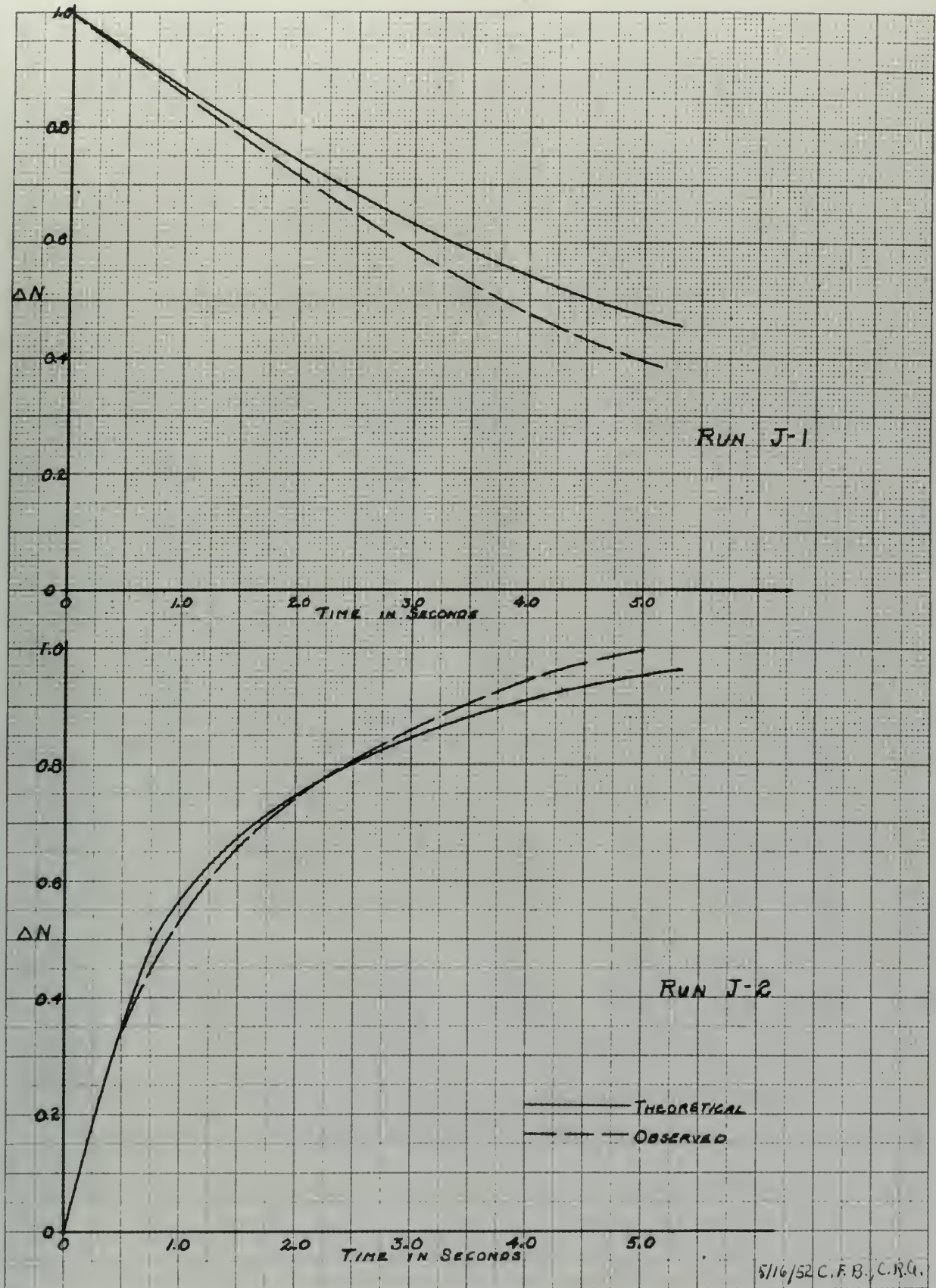


FIG 16



SPEED TRANSIENT FOR STEP CHANGE IN FIRING ANGLE
IGNITRON MOTOR CONTROL

FIG 17



SPEED TRANSIENT FOR STEP CHANGE IN FIRING ANGLE
IGNITRON MOTOR CONTROL

FIG 18

TABLE III

OPERATING CONDITIONS AND CIRCUIT PARAMETERS FOR
OBSERVED AND PREDICTED SPEED TRANSIENTS FIGURES 14 to 18

Values given based on full scale motor.

Run	Speed in rpm		Firing Angle		R_m in ohms.	i_3 amps.	R_a ohms.	T
	Initial	Final	Initial	Final				
J-1	902	320	96	135	111	2.0	52.0	6.7
J-2	460	742	127	109	111	2.0	6.2 10.4	1.12 1.81
K-1	205	430	137	121	26.9	2.8	7.3	1.10
K-2	728	465	97	120	26.9	2.8	12.4	1.62
L-1	490	210	106	132	9.6	2.8	18.2	1.20
L-2	365	590	118	97	9.6	2.8	3.1	0.45
N-1	408	825	104	62	5.2	2.8	0.57	0.10
N-2	830	388	60	108	5.2	2.8	6.7	0.56
P-1	715	885	70	52	4.0	3.9	0.57	0.96
P-3	880	680	52	74	4.0	3.9	2.8 0.48	0.31 0.096
Q-1	535	750	85	65	3.2	5.3	0.48	0.092
Q-2	875	778	51	63	3.2	5.3	0.48	0.092

Values for speeds, firing angles, R_m and i_3
taken from reference 4.

TABLE III

OPERATING CONDITIONS AND CIRCUIT PARAMETERS FOR
OPERATED AND MODIFIED STARK-WHEELER TUBES 14 to 18

Values given based on full scale meter.

Run	Initial Temp	Speed in rpm	Initial Temp	Initial Temp	Initial Temp	Initial Temp	Initial Temp
1-1	202	320	25	112	111	2.0	22.0
1-2	400	742	152	102	111	2.0	10.2
E-1	202	430	122	121	20.2	2.8	7.3
K-2	720	402	22	120	20.2	2.0	12.2
I-1	400	210	100	122	2.0	2.8	12.2
I-2	202	200	112	22	2.0	2.8	7.1
M-1	400	822	102	22	2.2	2.8	0.22
M-2	830	300	20	100	2.2	2.8	0.22
E-1	712	882	20	22	4.0	3.2	0.22
E-2	850	680	22	22	4.0	3.2	0.22
E-1	222	720	22	22	3.2	2.2	0.22
E-2	222	720	22	22	3.2	2.2	0.22

Values for speed, firing angle, θ and ϕ
taken from reference 4.

ANALYTICAL DETERMINATION OF SPEED-TORQUE CURVES

In order to predict the speed transients for the ignitron-motor system by the methods previously described, it is necessary to obtain the speed-torque or counter-emf versus armature current curves of the motor for various ignitron firing angles. These may be obtained experimentally from the actual motor supplied by the ignitron rectifier from readings of direct current and voltage at the motor terminals corrected for the voltage drop through the armature resistance, or they may be obtained directly from an analogue circuit as described on pages 10 and 17.

An alternative method is available, however, for obtaining these curves directly from the machine constants of the d-c motor and the ignitron rectifier characteristics. For power applications in which a multi-phase rectifier is used, there will be two distinct regions to the performance curves, one for discontinuous conduction and the other for continuous conduction.

The phenomenon of discontinuous conduction where no current flows during portions of each cycle has been discussed in detail by Vedder and Puchlowski¹⁾ in connection with single-phase full-wave rectifiers. The continuous conduction condition where one of the rectifier tubes fires at the instant the tube in the preceding phase cuts off, and where the instantaneous current to the motor does not go to zero during any portion of the cycle has not been considered in such detail.

ANALYSIS OF THE OPERATION OF THE MOTOR

In order to predict the speed of the motor for the various motor systems by the method previously described, it is necessary to obtain the speed-torque or constant-torque curves. The current curves of the motor for various firing angles. These may be obtained experimentally from the actual motor supplied by the limiter rectifier from readings of direct current and voltage at the motor terminals corrected for the voltage drop through the circuit resistance, or they may be obtained directly from an analogous circuit as described on pages 16 and 17.

An alternative method is available, however, for obtaining these curves directly from the machine constants of the d-c motor and the limiter rectifier characteristics. For power applications in which a multi-phase rectifier is used, there will be two distinct regions in the motor's operation, one for discontinuous conduction and the other for continuous conduction.

The phenomenon of discontinuous conduction exists in current flow during portions of each cycle and has been discussed in detail by ¹⁾ Sadler and Peterson. The continuous conduction single-phase full-wave rectifier. The continuous conduction condition exists when one of the rectifier tubes fires at the instant the tube in the preceding phase cuts off, and when the instantaneous current to the motor does not go to zero during any portion of the cycle has not been maintained in each detail.

Continuous Conduction.

For continuous conduction in the steady state and an arbitrary firing angle, the voltage applied to the motor terminals has the form shown in Figure 19. This may be compared with the oscillograms of Figure 4.

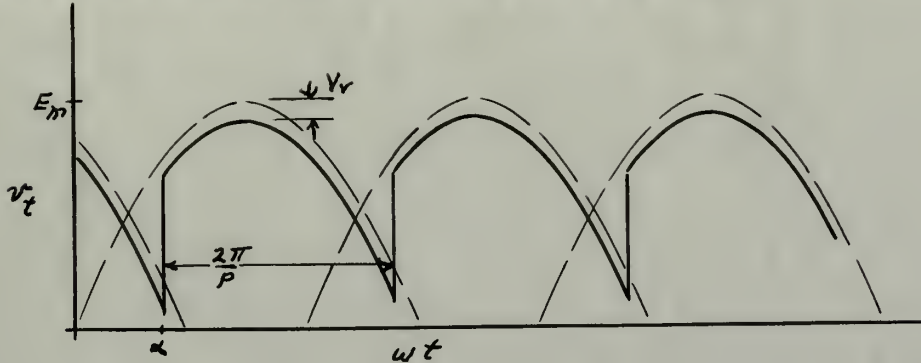


Figure 19. Motor Terminal Voltage for Continuous Conduction.

For a rectifier of p symmetrical phases, each tube is conducting over an interval of $\frac{2\pi}{p}$ and hence the applied voltage over one interval is given by

$$v_t = E_m \sin \omega t - V_r$$

$$\text{for } \alpha < \omega t < \alpha + \frac{2\pi}{p}$$

where E_m = peak value of applied voltage to rectifier.

ω = 2π x line frequency.

V_r = rectifier tube voltage drop.

Since the phases are symmetrical, the applied voltage over each interval is the same and the average value of v_t over one interval is the direct voltage component applied to the motor. This may be found from

Continuous Conduction

For continuous conduction in the steady state and an arbitrary firing angle, the voltage applied to the motor terminals has the form shown in Figure 13. This may be compared with the waveforms of Figure 4.

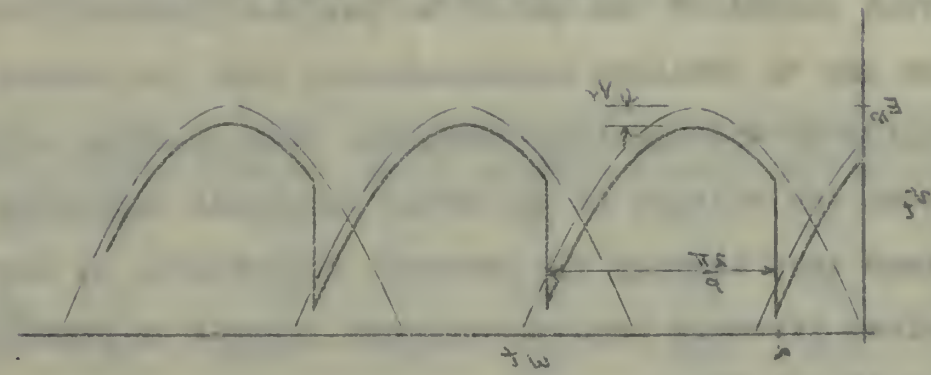


Figure 13. Motor terminal voltage for continuous conduction.

For a freewheeling diode, the average value of the voltage over one interval of $\frac{\pi}{\omega}$ and hence the applied voltage over one interval is given by

$$V_a = \frac{1}{\pi} \int_0^\pi V_m \sin(\omega t - \alpha) d(\omega t)$$

$$\text{for } \alpha < \omega < \frac{5\pi}{2}$$

where V_a = average value of applied voltage to freewheeling diode.
 $\omega = 2\pi \times \text{frequency}$.
The freewheeling diode voltage drop is neglected.
Since the freewheeling diode is ideal, the applied voltage over each interval is the same and the average value of V_a over one interval is the same as the average value of V_a over the motor. This may be found from the waveform in Figure 13 and is given by

$$\begin{aligned}
 V_t &= \frac{p}{2\pi} \int_{\alpha}^{\alpha + \frac{2\pi}{p}} (E_m \sin \omega t - V_r) d(\omega t) \\
 &= \frac{p}{2\pi} E_m \left[\cos \alpha - \cos \left(\alpha + \frac{2\pi}{p} \right) \right] - V_r \\
 &= \frac{p}{2\pi} E_m \left[\left(\cos \alpha - \cos \alpha \cos \frac{2\pi}{p} + \sin \alpha \sin \frac{2\pi}{p} \right) \right] - V_r \\
 &= p \frac{E_m}{2\pi} \left[\left(\cos \alpha (2 \sin^2 \frac{\pi}{p}) + \sin \alpha (2 \sin \frac{\pi}{p} \cos \frac{\pi}{p}) \right) \right] - V_r \\
 &= p \frac{E_m \sin \pi/p}{\pi} (\sin \alpha + \pi/p) - V_r \quad (27)
 \end{aligned}$$

For continuous conduction, therefore, V_t with a given rectifier is a function only of the firing angle; and if the motor is assumed to have linear electrical characteristics, the counter-emf versus armature current relationship for each firing angle is given by the ordinary d-c motor equation.

$$E_b = V_t - I_a R_a - V_b \quad (28)$$

where R_a = armature resistance of the motor

V_b = motor brush drop

The continuous conduction portion of the curves are thus a series of straight lines and can be plotted knowing only the effective armature resistance and the brush drop.

Discontinuous Conduction - Exact Method.

The shape of the curves where conduction is discontinuous may be determined accurately if required from the relationships derived by Vedder and Luchlowski¹⁾; Harris⁵⁾ and Heller⁴⁾.

-
5. Harris, L. D., "Servomechanism Characteristics of D-C Motor Driven by Controlled Rectifiers", IEEE Technical Paper 51-297, July 1951.

$$V_c = \frac{2}{\pi} \left[\cos \alpha - \cos \left(\alpha + \frac{\pi}{2} \right) \right] = \frac{2}{\pi} \left[\cos \alpha - \cos \alpha \cos \frac{\pi}{2} + \sin \alpha \sin \frac{\pi}{2} \right] = \frac{2}{\pi} \left[\cos \alpha - \cos \alpha \cdot 0 + \sin \alpha \cdot 1 \right] = \frac{2}{\pi} \sin \alpha$$

$$V_c = \frac{2}{\pi} \sin \alpha$$

For continuous conduction, the average value of the firing angle is a function of the firing angle; and as the motor is assumed to have linear electrical characteristics, the average value of the firing angle is a function of the firing angle. The firing angle is given by the ordinary d-c motor equation.

$$V_c = V - I_a R_a - I_a R_c$$

where R_a = armature resistance of the motor
 R_c = motor brush drop

The continuous conduction portion of the curves are thus a series of straight lines and can be plotted knowing only the effective armature resistance and the brush drop.

Discontinuous Conduction - Speed Control

The shape of the curves where conduction is discontinuous may be determined numerically by applying the relation: $V_c = V - I_a R_a - I_a R_c$ and solving for I_a .

$$E_b = E_m \cos \theta_a \frac{(\sin(\alpha + \theta_a + r) - \epsilon \frac{-r}{\tau_a} \sin(\alpha + \theta_a)) - V_r - V_b}{1 - \epsilon - r/\tau_a} \quad (29)$$

$$I_a = \frac{E_m}{R_a} \frac{2\pi}{p} \left[(\cos \alpha - \cos(\alpha + r) - \frac{E_b + V_r + V_b}{E_m} r) \right] \quad (30)$$

where $\theta_a = -\tan^{-1} \tau_a$

$$\tau_a = \frac{\omega L_a}{R_a}$$

r = angle during which conduction takes place

$$(r < \frac{2\pi}{p})$$

By taking values of α and r , E_b and I_a may be found to determine the desired points on the discontinuous portions of the performance curves. For the boundary condition between continuous and discontinuous conduction at a particular firing angle, $r = \frac{2\pi}{p}$, so that equation (30) reduces to the combination of equations (27) and (28), and equation (29) becomes

$$E_b = E_m \cos \theta_a \frac{(\sin(\alpha + \theta_a + \frac{2\pi}{p}) - \epsilon \frac{-\frac{2\pi}{p}}{\tau_a} \sin(\alpha + \theta_a)) - V_r - V_b}{1 - \epsilon - \frac{2\pi}{p\tau_a}} \quad (31)$$

Discontinuous Conduction - Approximate Method.

Using equations (29) and (30) involves considerable tedious calculation. However, it was found possible to approximate the discontinuous conduction performance curves for the system studied by assuming them to be simple parabolas of the form

$$I_a = k' (E_b \text{ max} - E_b)^2 \quad (32)$$

$$\text{where } E_b \text{ max} = E_m - V_r - V_b \quad (33)$$

$$\text{for } \alpha \leq 90^\circ$$

$$\text{and } E_b \text{ max} = E_m \sin \alpha - V_r - V_b \quad (34)$$

$$\text{for } \alpha > 90^\circ$$

$$I_D = \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - \cos(\alpha + \theta)}{1 - \cos(\alpha - \theta)} d\theta \quad (25)$$

$$I_D = \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - \cos(\alpha + \theta)}{1 - \cos(\alpha - \theta)} d\theta \quad (26)$$

$$\cos \theta = -\cos \alpha$$

$$\theta = \pi - \alpha$$

α = angle between which observation lines pass

$$\left(\frac{\pi}{2} < \alpha \right)$$

By taking values of α and θ , one can find the

determine the desired points on the line of sight between
of the observation curves. For the boundary condition between
continuous and discontinuous observation of a particular line
angle, $\alpha = \frac{\pi}{2}$, so that equation (25) reduces to the value
tion of equations (27) and (28), and equation (29) becomes

$$I_D = \frac{1}{2\pi} \int_0^{2\pi} \frac{1 - \cos(\alpha + \theta)}{1 - \cos(\alpha - \theta)} d\theta \quad (31)$$

Minimum observation - maximum error.

Using equations (25) and (26) the values of I_D are

calculated. However, it was found possible to approximate
the continuous observation curves for the
system modeled by assuming them to be single points of the

form

$$I_D = 1 - \cos(\alpha - \theta)$$

$$I_D \max = 1 - \cos \alpha \quad (32)$$

$$\alpha \leq 90^\circ$$

$$I_D \max = 1 - \cos \alpha \quad (33)$$

The constant k' is different for each firing angle and was determined by the fact that the discontinuous and continuous conduction curves for a given firing angle intersect at the point evaluated from equation (31).

The curves plotted on Figure 2 for firing angles up to 110° were obtained by this method and indicate very close agreement with the experimentally determined points shown. Since there can be no continuous conduction for firing angles of 120° or more with a three phase rectifier, the constant k' can not be evaluated under these conditions. Accordingly, the experimental points were used in drawing the curves for 130° and 150° on Figure 2. However, since k' appears to change slowly at large angles, these could have been obtained with some sacrifice in accuracy by assuming k' to have the same value as at 110° . This would probably be sufficiently accurate for most use, since this area of the curves is of little practical interest.

While it is obvious that this is a purely empirical method of obtaining the discontinuous conduction characteristics, it involves relatively simple calculations and is considered to be practicable for three-phase rectifiers used with motors having the same general characteristics as the unit used for this investigation. The end points for the curves are theoretically correct; but the actual shape of the intervening portion as given by equations (29) and (30) is a function primarily of the reactance to resistance ratio

The constant k is different for each liquid and was determined by the fact that the displacement and position of the curve are given by the same interest at the point indicated by equation (31).

The curves plotted on figures 4 and 5 are taken up to 110° were obtained by this method and indicate very close agreement with the experimentally determined points shown. Since there can be no displacement occurring for liquid values of 120° or more with a three phase system, the constant k can not be evaluated under these conditions. Accordingly, the experimental points were used in plotting the curves for 130° and 140° on figure 5. However, since k appears to change slightly at these points, these points were also plotted when more variation in density is observed in figure 5. To have the same value as at 110° , the value would probably be sufficiently accurate for most use, since this was at the lowest of little practical interest.

While it is obvious that this is a fairly complicated method of obtaining the displacement constant and the latent heat, it involves relatively simple calculations and is considered to be practicable for three-phase systems used with water having the same general characteristics as the unit used for this investigation. The unit values for the curves are numerically correct; and the actual shape of the interesting portion as shown in figures 4(a) and 5(a) is a function primarily of the temperature of condensation ratio

of the motor armature circuit. The approximation presented here has been shown to be valid for one particular ratio and may not be entirely applicable to motors with different characteristics.

Effective Armature Resistance.

Determining the effective value of the armature resistance to be used in obtaining theoretical performance curves by any of the above methods is one of the major practical problems. As previously noted, the 60 cycle a-c resistance was used throughout this investigation; and although the justification for it is questionable, the theoretical and experimental results obtained show close agreement.

of the motor apparatus circuit. The apparatus presented here has been shown to be valid for the physiological tests and may not be entirely applicable to groups with different

characteristics.

Effective Pressure Measurement

Determining the effective value of the structure weight-
ness is needed in obtaining essential physiological values
by any of the above methods is one of the major practical
problems. As previously noted, the 60 cycle/sec resistance
was used throughout this investigation; and although the
justification for it is questionable, the satisfactory and
experimental results obtained are also apparent.

CONCLUSIONS

Analogue Circuit.

The analogue was originally proposed as a means of obtaining the steady state counter-emf versus armature current curves and for testing feedback arrangements to improve the transient speed response.

The results of the tests with the analogue circuit indicate that it represents the dynamic and steady-state behavior of the ignitron motor control system for all conditions except those involving operation in or near the boundary region between continuous and discontinuous conduction. However, since the operation of the circuit is actually unstable under these conditions, and since this region extends over a considerable portion of the characteristics, the usefulness of the analogue is limited.

It is not considered suitable for determining accurate steady-state characteristics, since the areas where the analogue does not give consistent results are the only ones of interest which cannot be easily obtained analytically.

Similarly, it is not considered practical for general use in feedback analysis because of its inherent instability over an appreciable portion of its operating range. However, it could be used for certain types of feedback investigations where stability is not involved.

CONCLUSION

General Remarks

The analysis was originally proposed as a means of obtaining the steady state equilibrium values of the various parameters and the resulting feedback characteristics to improve the transient speed response. The results of the tests with the various circuits indicate that it is possible to obtain the dynamic and steady-state behavior of the system under control system for all conditions except those involving operation in or near the boundary region between continuous and discontinuous conduction. However, since the operation of the circuit is actually unstable under these conditions, and since this region extends over a considerable portion of the parameter space, the usefulness of the analysis is limited. It is not recommended suitable for determining accurate steady-state characteristics, since the errors under the analysis does not give consistent results for the only cases of interest which would be really significant analysis.

Summary

Similarly, it is not recommended practical for general use in feedback analysis because of its inherent instability over an appreciable portion of its operating range. However, it could be used for certain types of feedback investigations where stability is not involved.

For the overall transient analysis problem, the method of predicting the response from a time constant derived from the steady-state characteristics of the system is considered easier and more practical for engineering investigations than either the use of an analogue circuit or the step by step analytical methods developed by Heller⁴⁾. By using the approximate method for obtaining the steady-state characteristics for the system, this method reduces to a relatively simple analysis which nevertheless yields results of sufficient accuracy for nearly all purposes.

For the overall treatment analysis, the method of predicting the response from a time constant derived from the steady-state observation of the system is considered easier and more practical for engineering investigations than either the use of an analogue circuit or the step by step analytical methods developed by Heller⁽⁵⁾. By using the approximate method for obtaining the steady-state characteristics for the system, this method reduces to a relatively simple analysis while having almost equal results of sufficient accuracy for many all purposes.

RECOMMENDATIONS

In spite of its limitations, the use of the analogue circuit is recommended for investigations leading toward the use of dynamic braking³⁾ to reduce the excessive slow-down times now required for an ignitron-fed motor.

If the analogue circuit is set up for this purpose, it is recommended that a high resistance be placed across the rectifier output with a view toward reducing the effects of the undesirable current cut-off transient encountered in this investigation.

For a determination of the servomechanisms characteristics of the ignitron control system, it is recommended that the work of Harris⁵⁾ be extended, using the concepts developed in this paper for the prediction of transient response, to include the condition encountered with three-phase rectifier where the speed-up and slow-down behavior are in general appreciably different.

In any theoretical or analogue circuit work with a rectifier-controlled d-c motor, the assumed value of the effective resistance of the armature of the motor to the average or direct component of the armature current has considerable influence on the results obtained. Since the d-c and a-c resistance of most motors differ so widely, it is recommended that a theoretical and experimental study be made to determine the effective value of armature resistance under these conditions and if necessary a practical means of obtaining it for a given machine.

CONCLUSIONS

In spite of the limitations, the use of the analogue circuit is recommended for investigations leading toward the use of dynamic braking²⁾ to reduce the excessive slow-down time now required for an induction motor.

If the analogue circuit is not used for this purpose, it is recommended that a high resistance be placed across the resistor output with a view toward reducing the effects of the inductive current and its transient associated in this investigation.

For a determination of the recommended values of the values of the induction control system, it is recommended that the work of Harris²⁾ be extended, using the concepts developed in this paper for the prediction of transient response, to include the condition associated with three-phase rectifier where the speed-up and slow-down behavior are in general appreciably different.

In any theoretical or analogue circuit work with a rectifier-controlled d-c motor, the assumed value of the effective resistance of the inductor of the motor in the series or direct component of the resistor current has considerable influence on the results obtained. Since the d-c and a-c resistance of most motors differ so widely, it is recommended that a theoretical and experimental study be made to determine the effective value of resistance for use under these conditions and if necessary a practical means of obtaining it for a given machine.

BIBLIOGRAPHY

1. Vedder, E. H. and Fucalowski, K. P., "Theory of Rectifier D-C Motor Drive", AIEE Trans., 62, 1943, pages 863-870.
2. Schmidt, A. and Smith, W. P., "Operation of Large D-C Motors from Controlled Rectifiers", AIEE Trans., 67, 1948, pages 679-683.
3. Chute, G. M., "Electronic Motor and Welder Controls", McGraw-Hill Book Co., 1951, pages 191-202, 226-277.
4. Heller, P. N., "Transient Speed and Armature Current Characteristics of an Ignitron-Fed D-C Motor", M.I.T. E.E. Dept. Thesis, 1951.
5. Harris, L. D., "Servomechanism Characteristics of D-C Motor Driven by Controlled Rectifiers", AIEE Technical Paper 51-297, July 1951.

REFERENCES

1. Webster, E. W. and Van Dine, J. L., "Theory of
Controlled U-2 Motor Drive", AIEE Trans., 62, 1945,
pages 803-810.
2. Schmidt, A. and Smith, E. L., "Operation of Large
U-2 Motors from Controlled Rectifiers", AIEE Trans.,
62, 1945, pages 810-813.
3. Chute, G. E., "Electronic Motor and Field Controls",
McGraw-Hill Book Co., 1951, pages 192-202, 226-234.
4. Miller, E. W., "Transistor Speed and Torque Control
Characteristics of an Inverter-Feed U-2 Motor", M.I.T.
M.S. Dept. Thesis, 1951.
5. Morris, L. U., "Synchronous Rectification of
U-2 Motor Drive by Controlled Rectifiers", AIEE
Technical Paper 51-107, July 1951.

APPENDIX

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APPENDIX A

Nameplate data of motor and ignitron rectifier represented by the analogue circuit.

MOTOR

M.I.T. No. 250

Westinghouse

Type Sk, Frame 93	Style SC 1168256
15 horsepower	Serial 5SSC 1168256
56 amperes	230 volts D. C.
40°C Rise, continuous duty	850 rpm

GENERATOR

M.I.T. No. 251

Westinghouse

Type Sk, No. 90	Style 153133 B
18 horsepower	Serial no. 1556940
67 amperes	230 volts D. C.
780-1500 rpm	

RECTIFIER

M.I.T. No. 147

Westinghouse

Ignitron Rectifier

Input

18.75 kw 230 volt 60 cycle 3 phase

Output

230 volt 75 amperes D.C.

Serial 11P778

APPENDIX A

Manufacture date of motor and ignition resistor register
sent by the engine electric.

NOTES

M.I.T. No. 250

TESTING

Type SR, frame 93
15 horsepower
50 volts
4000 rpm, continuous duty
M.I.T. No. 250

TESTING

M.I.T. No. 251

TESTING

Type SR, No. 90
15 horsepower
60 volts
750-1500 rpm
M.I.T. No. 251

TESTING

M.I.T. No. 252

TESTING

Ignition Resistor

Notes

18.75 hp 250 volt 60 cycle 3 phase

Notes

250 volt 75 horsepower D.C.

Serial 21775

Thesis
B829

Bryant

17139

Investigation of transients in an analogue circuit for an ignitron motor control system.

Thesis
B829

Bryant

17139

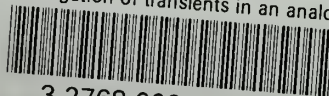
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Investigation of transients in an analog



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